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6.1: Factoring Out the Greatest Common Factor Factor out the Greatest Common Factor when it is a common monomial expression Factor out the Greatest Common Factor when it is a common parenthetical expression Factors are the building blocks of multiplication. They are the numbers that you can multiply together to produce another number: [latex]2[/latex] and [latex]10[/latex] are factors of [latex]20[/latex], as are [latex]4, 5, 1, 20[/latex]. To factor a number is to rewrite it as a product. [latex]20=4\cdot{5}[/latex] or [latex]20=1\cdot{20}[/latex]. In algebra, we use the word factor as both a noun – something being multiplied – and as a verb – the action of rewriting a sum or difference as a product. Factoring is very helpful in simplifying expressions and solving equations involving polynomials. The greatest common factor (GCF) of two numbers is the largest number that divides evenly into both numbers. For instance, [latex]4[/latex] is the GCF of [latex]16[/latex] and [latex]20[/latex] because it is the largest number that divides evenly into both [latex]16[/latex] and [latex]20[/latex]. The GCF of polynomials works the same way: [latex]4x[/latex] is the GCF of [latex]16x[/latex] and [latex]20{x}^{2}[/latex] because it is the largest polynomial that divides evenly into both [latex]16x[/latex] and [latex]20{x}^{2}[/latex]. When factoring a polynomial expression, our first step should be to check for a GCF. Look for the GCF of the coefficients, and then look for the GCF of the variables. The greatest common factor (GCF) of a group of given monomials is the largest* monomial that divides evenly into the polynomials. *Let us clarify what is meant by the "largest" monomial. For the coefficient, we would choose the GCF of the coefficients of all given monomials. And for any variables, the GCF must have the largest degree possible. How to actually determine this is demonstrated in the example below. In the example above, the monomials have the factors [latex]5[/latex], b, and b in common, which means their greatest common factor is [latex]5\cdot{b}\cdot{b}[/latex], or simply [latex]5b^{2}[/latex]. The video that follows gives an example of finding the greatest common factor of two monomials with only one variable. Sometimes you may encounter a polynomial with more than one variable, so it is important to check whether both variables are part of the GCF. In the next example, we find the GCF of two terms which both contain two variables. The video that follows shows another example of finding the greatest common factor of two monomials with more than one variable. You might be picking up on a useful shortcut for the variables. Notice that – assuming the variable is in common – we always select the smaller power of the variable. Now that you have practiced identifying the GCF of terms with one and two variables, we can apply this idea to factoring the GCF out of a polynomial. Notice that the instructions are now "Factor" instead of "Find the greatest common factor." First, you must identify the greatest common factor of the terms. You can then use the distributive property to rewrite the polynomial in factored form. Forward: Product of a number and a sum: [latex]a\left(b+c\right)=a\cdot{b}+a\cdot{c}[/latex]. You can say that "[latex]a[/latex] is being distributed over [latex]b+c[/latex]." Backward: Sum of the products: [latex]a\cdot{b}+a\cdot{c}=a\left(b+c\right)[/latex]. Here you can say that "[latex]a[/latex] is being factored out." We first learned that we could distribute a factor over a sum or difference, now we are learning that we can "undo" the distributive property with factoring. The factored form of the polynomial [latex]25b^{3}+10b^{2}[/latex] is [latex]5b^{2}\left(5b+2\right)[/latex]. You can check this by doing the multiplication. [latex]5b^{2}\left(5b+2\right)=25b^{3}+10b^{2}[/latex]. Note that if you do not factor the greatest common factor at first, you can continue factoring, rather than start all over. For example: [latex]\begin{array}{l}25b^{3}+10b^{2}=5\left(5b^{3}+2b^{2}\right)\,\,\,\,\,\,\,\,\,\,\,\,\text{Factor out }5\\\,=5b^{2}\left(5b+2\right) \,\,\,\,\,\,\,\,\,\,\,\,\,\,\,\,\text{Factor out }b^{2}\end{array}[/latex] Notice that you arrive at the same simplified form whether you factor out the GCF immediately or if you pull out factors individually. In the following video, we show two more examples of how to find and factor the GCF from binomials. We will show one last example of finding the GCF of a polynomial with several terms and two variables. No matter how large the polynomial, you can use the same technique described below to factor out its GCF. How To: Given a polynomial expression, factor out the greatest common factor Identify the GCF of the coefficients. Identify the GCF of the variables. Write together to find the GCF of the expression. Determine what the GCF needs to be multiplied by to obtain each term in the expression. Write the factored expression as the product of the GCF and the sum of the terms we need to multiply by. Factor out the GCF: [latex]6{x}^{3}{y}^{3}+45{x}^{2}{y}^{2}+21xy[/latex]. In the following video, you will see two more example of how to find and factor our the greatest common factor of a polynomial. In the next example, the leading coefficient is negative. In this case, it is convention to include the negative with the GCF. In addition, we will see in future sections that this can assist us in other ways as well. Factor out the GCF: [latex]-24x^{8}+32x^{5}[/latex] Another interesting situation we may encounter is where the GCF could even include an entire expression containing multiple terms, as shown in the next example. Factor out the GCF: [latex]3x^{2}(2x-5)+7(2x-5)[/latex] This last example, as well as the overall idea of factoring out the GCF, will be of great use in the next section. When we study fractions, we learn that the greatest common factor (GCF) of two numbers is the largest number that divides evenly into both numbers. For instance, [latex]4[/latex] is the GCF of [latex]16[/latex] and [latex]20[/latex] because it is the largest number that divides evenly into both [latex]16[/latex] and [latex]20[/latex] The GCF of polynomials works the same way: [latex]4x[/latex] is the GCF of [latex]16x[/latex] and [latex]20{x}^{2}[/latex] because it is the largest polynomial that divides evenly into both [latex]16x[/latex] and [latex]20{x}^{2}[/latex]. When factoring a polynomial expression, our first step should be to check for a GCF. Look for the GCF of the coefficients, and then look for the GCF of the variables. The greatest common factor (GCF) of polynomials is the largest polynomial that divides evenly into the polynomials. How To: Given a polynomial expression, factor out the greatest common factor. Identify the GCF of the coefficients. Identify the GCF of the variables. Combine to find the GCF of the expression. Determine what the GCF needs to be multiplied by to obtain each term in the expression. Write the factored expression as the product of the GCF and the sum of the terms we need to multiply by. Factor [latex]6{x}^{3}{y}^{3}+45{x}^{2}{y}^{2}+21xy[/latex]. First, find the GCF of the expression. The GCF of [latex]6,45[/latex], and [latex]21[/latex] is [latex]3[/latex]. The GCF of [latex]{x}^{3},{x}^{2}[/latex], and [latex]x[/latex] is [latex]x[/latex]. (Note that the GCF of a set of expressions in the form [latex]{x}^{n}[/latex] will always be the exponent of lowest degree.) And the GCF of [latex]{y}^{3},{y}^{2}[/latex], and [latex]y[/latex] is [latex]y[/latex]. Combine these to find the GCF of the polynomial, [latex]3xy[/latex]. Next, determine what the GCF needs to be multiplied by to obtain each term of the polynomial. We find that [latex]3xy\left(2{x}^{2}{y}^{2}\right)=6{x}^{3}{y}^{3},3xy\left(15xy\right)=45{x}^{2}{y}^{2}[/latex], and [latex]3xy\left(7\right)=21xy[/latex]. Finally, write the factored expression as the product of the GCF and the sum of the terms we needed to multiply by. [latex]\left(3xy\right)\left(2{x}^{2}{y}^{2}+15xy+7\right)[/latex] After factoring, we can check our work by multiplying. Use the distributive property to confirm that [latex]\left(3xy\right)\left(2{x}^{2}{y}^{2}+15xy+7\right)=6{x}^{3}{y}^{3}+45{x}^{2}{y}^{2}+21xy[/latex]. Factor [latex]x\left({b}^{2}-a\right)+6\left({b}^{2}-a\right)[/latex] by pulling out the GCF. Solution The factors that can divide any number one or more than one without leaving any remainder are termed as a common factor. After the factorization, when we compare the factors of two or more we will get that some of the factors are the same or common and those factors are known as common factors.What are Common Factors?When a number is divided by a divisor, that divisor is known as a factor of that number. A factor cannot be greater than the given number but can be less than or equal to the given number. But 1 is the common factor of every number and every number is a factor of itself. Therefore, if it is a multiple of that divisor then the number may be divided perfectly by many divisors.For Example The factors of55 =1,5,1145 = 1,3,5,9,15,45here, 1 and 5 are the numbers that are perfectly dividing both the numbers. Thus the common factors of 55 and 45 are 1 and 5."When two or more numbers are exactly divided by the same number(s), then the common divisors of the given numbers are known as' common factors." A common factor is also defined as a number that divides two or more numbers exactly without leaving any remainder.How to Find Common FactorsTo find the common factor of two or more numbers, follow the steps below:i) List the factors of each number.ii) Compare the factors of every number.iii) List the same (i.e. common) factors in every number. These factors are called common factors of the given numbers.Greatest Common FactorWhen the factorization of one or more numbers has been calculated then the greatest common factor of the given numbers is termed as The GCF-Greatest Common Factor. After that, there are some factors that are common in the given numbers. Out of these numbers, the number which is the largest is the greatest common factor.Let's suppose that m and n are natural numbers.The GCF of two natural numbers m and n is the largest possible number that divides m and n. It is also known as the HCF-Highest Common Factor or (GCD) greatest common divisor.Example 1: Find the common factors of 48 and 68 and then find out the greatest common factor between them?Solution: Factors of 24 : 1, 2, 3, 4, 6, 8, 12, 24 Factors of 68 : 1, 2, 4, 17, 34 and 68.Thus, the common factors of 24 and 68 are 1, 2, and 4. Out of these common factors, 4 is the Greatest common factor. Therefore , the Greatest Common Factor of 24 and 68 is 4. It can be written as GCF(24, 68) = 4.Example 2: Explain How to find the common factors?Solution:The factors are the numbers that are a number's exact divisors. There are some steps to take in order to identify the common factors.Step 1 : Separately write down all the factors of the given numbers.Step 2 : Now look for the factors that are common in the given numbers and write them down in a separate row.Example : Find out the common factors of 8 and 16 ?Solution : Factors of 8 are 1,2,4,8 Factors of 16 are 1,2, 4,8,16 so common factors of 8 and 16 are 1,2,4,8Sample ProblemsProblem 1: Find out the common factors of 17 and 21?Solution:The factors of 17 and 21 are,Factors of 17 = 1, 171 x 17 = 17 17 x 1 = 17 Factors of 21 = 1,3, 7, 211 x 21 = 21 3 x 7 = 217 x 3 = 21 21 x 1 = 21Thus, the common factor of 17 and 21 is 1. Problem 2: Find the common factors of 56 and 36.Solution:The factors of 56 and 36 are,Factors of 56 are1 x 56 = 56 2 x 28 = 56 4 x 14 = 567 x 8 = 56 { 1, 2, 4, 7, 8, 14, 28, 56 }Similarly for the Factors of 36 = 1, 2, 3, 4, 6, 9, 12, 18, and 36Thus, the common factors of 56 and 36 are 1, 2, 4 Problem 3: Find the common factor of 16, 28, and 42. What will be the greatest common factor?Solution:The factors of the three numbers- 16, 28, and 42 areFactors of 16 = 1, 2, 4, 8, and 16Factors of 28 = 1, 2, 4, 7, 14, and 28Factors of 42 = 1, 2, 3, 6, 7, 14, 21 and 42The common factors of 16, 28, and 42 are 1, 2 . So, the greatest common factor of 16, 28, and 42 is 2 . GCF(16,28,42) = 2Problem 4: What are the common factors between 55 and 25? Find out the GCF?Solution: The factors of 55 are 1 , 5 , 11 and 55 The factors of 25 are 1, 5 and 25 .Thus, the common factor of 55 and 25 are 1 and 5 here the greatest common factor is 5GCF(55, 25) = 5Problem 5: What are the common factors of 10 and 15?Solution:The factors of 10 are: 1, 2, 5, 10The factors of 15 are: 1, 3, 5, 15Therefore, the common factors of 10 and 15 are 1 and 5.Problem 6: What are the common factors of 18 and 19? Find out the greatest common factor?Solution:The factors of 18 are : 1, 2, 3, 6, 9 and 18The factors of 19 are : 1, 19Therefore, the common factors of 18 and 19 are only 1 so it is only common factor or highest factor.Practical Applications of Common Factors and GCFThe concept of common factors and the greatest common factor (GCF) has practical applications in various areas of mathematics, everyday problem-solving, and even in certain real-world situations. Some of the practical applications includeSimplifying FractionsWhen simplifying fractions, the GCF of the numerator and denominator is used to reduce the fraction to its simplest form.Factoring Polynomials In algebra, the GCF is used to factor out the greatest common factor from polynomial expressions.Problem Solving in Number TheoryThe GCF is used in number theory problems, such as finding whether two numbers are coprime (if their GCF is 1).Dividing Items into GroupsWhen dividing items into groups or making equal distributions, the GCF helps in determining the largest possible equal groups.Simplifying RatiosGCF is used to simplify ratios, which is important in various practical applications, such as cooking, construction, and financial planning.Finding LCM (Least Common Multiple) Using GCFThe GCF used along with the product of two numbers to find the least common multiple (LCM). The formula for finding LCM isLCM = Product of the numbers/ GCFReducing Exponents in ExpressionsTo simplify the expressions with exponents, the GCF helps to reduce the exponents.Practice Problems1.Find the common factors of the following pairs of numbers:a) 12 and 18b) 20 and 50c) 45 and 602. Find the GCF of the following pairs of numbers:a) 16 and 24b) 36 and 48c) 27 and 63Related Questions:What is the greatest common factor?The Greatest Common Factor is the largest of the common factors of two or more numbers.How to find common factors? To find the common factor of two or more numbers, i) List the factors of each number. ii) Compare the factors of every number. iii) List the same factors in every number. SummaryThe concepts of common factors and GCF are not just abstract mathematical ideas, but they are the tools that can be applied to make problem-solving more efficient and effective in various contexts. Download Article Follow our guide to determine the GCF of two numbers Download Article Finding the greatest common factor (GCF)[1] of a number set can be easy, but there are several steps you'll need to follow to get there. In order to find the greatest common factor of two numbers, you'll need to factor out both of those numbers using your knowledge of timetables, then identify the largest number that appears in both sets of factors. Factor out each number to the prime factors. Find the prime factors that each number has in common. Multiply the prime factors together to find the GCF. 1 Find factors of the number. You don't have to know prime factorization to find the greatest common factor. Start by finding all the factors of the set you are comparing.[2] 2 Compare the sets of factors until you find the biggest number that's in both sets. Advertisement 1 Factor each number completely into its prime numbers.[3] A prime number is number greater than 1 that has no factors but itself. Examples of prime numbers include 5, 17, 97, and 331, to name just a few. 2 Identify any common prime factors.[4] Pick out any prime numbers between the set that are the same. There can be several common factors, one common factor, or none. 3 Calculate: If there are no common factors then the greatest common factor is 1. If there's only one prime common factor, then that's your common factor. If there are multiple prime common factors, then multiply all the prime common factors together to get your greatest common factor[5] . 4 To demonstrate this method, study this example. Advertisement Add New Question Question What level of questions should I make for a test? Should I make the test easy, medium or hard? It depends upon your knowledge of your students. If you see that your students are very competitive and clever, you can make the test hard, so as to challenge their knowledge about the subject. If your students are beginners and have a lot to learn yet, making it easier will encourage them to keep learning more. Question What's the greatest common factor of 4x^3y, 8x^2y^3, xy^3z^5? The three terms are: $4x^3y$, $8x^2y^3$, and xy^3z^5. First, in terms of numerical coefficients, the lowest coefficient in the three terms is 1. The lowest x exponent is 1. The lowest y exponent is also 1. There is no z in two of the terms (so z is not a common factor). That means the greatest common factor among the three terms is 1xy (or xy). Question What is the use of prime numbers in our lives? The average person is never likely to use prime numbers. They do have certain applications within science and mathematics. See more answers Ask a Question Advertisement Thanks Thanks Advertisement This article was reviewed by Grace Imson, MA. Grace Imson is a math teacher with over 40 years of teaching experience. Grace is currently a math instructor at the City College of San Francisco and was previously in the Math Department at Saint Louis University. She has taught math at the elementary, middle, high school, and college levels. She has an MA in Education, specializing in Administration and Supervision from Saint Louis University. This article has been viewed 434,534 times. Co-authors: 31 Updated: July 24, 2024 Views: 434,534 Categories: Featured Articles | Multiplication and Division Article SummaryXTo find the greatest common factor of two or more numbers, make a list of all of the factors of each number. For example, for the number 10, the factors are 1, 2, 5, and 10, and for the number 21, the factors are 1, 3, 7, and 21. Then, compare the list of factors to find the largest number that the two have in common. For 10 and 21, the greatest common factor is 1. To learn more, like how to use prime numbers to find the greatest common factor, keep reading! Print Send fan mail to authors Thanks to all authors for creating a page that has been read 434,534 times. "I really love this! This page is really helpful; I learned many things. Thank you very much!" Share your story Year 8 Interactive Maths - Second Edition We know that: a(b + c) = ab + ac The reverse process, ab + ac = a(b + c), is called taking out the common factor. Consider the factorisation of the expression 5x + 15. Note that the common factor 5 has been taken out and placed in front of the brackets. The expression inside the brackets is obtained by dividing each term by 5. In general: To factorise an algebraic expression, take out the highest common factor and place it in front of the brackets. Then the expression inside the brackets is obtained by dividing each term by the highest common factor. Example 25 Factorise the following: Solution: Alternative way: Often, we set out the solution as follows: Note: The process of taking out a common factor is of great importance in algebra. With practice you will be able to find the highest common factor (HCF) readily and hence factorise the given expression. Example 26 Factorise the following: Solution: Note: We can check the answer by using the Distributive Law. Key Terms taking out a common factor The highest number that divides exactly into two or more numbers. It is the "greatest" thing for simplifying fractions! Let's start with an Example ... Find all the Factors of each number, Circle the Common factors, Choose the Greatest of those So ... what is a "Factor" ? Factors are numbers we can multiply together to get another number: A number can have many factors: Factors of 12 are 1, 2, 3, 4, 6 and 12 ... ... because 2 × 6 = 12, or 4 × 3 = 12, or 1 × 12 = 12. (Read how to find All the Factors of a Number. In our case we don't need the negative ones.) What is a "Common Factor" ? Say we have worked out the factors of two numbers: Factors of 12 are 1, 2, 3, 4, 6 and 12 Factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30 Then the common factors are those that are found in both lists: Notice that 1, 2, 3 and 6 appear in both lists? So, the common factors of 12 and 30 are: 1, 2, 3 and 6 It is a common factor when it is a factor of two (or more) numbers. Here is another example with three numbers: Factors of 15 are 1, 3, 5, and 15 Factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30 Factors of 105 are 1, 3, 5, 7, 15, 21, 35 and 105 The factors that are common to all three numbers are 1, 3, 5 and 15 In other words, the common factors of 15, 30 and 105 are 1, 3, 5 and 15 What is the "Greatest Common Factor" ? It is simply the largest of the common factors. In our previous example, the largest of the common factors is 15, so the Greatest Common Factor of 15, 30 and 105 is 15 The "Greatest Common Factor" is the largest of the common factors (of two or more numbers) Why is this Useful? One of the most useful things is when we want to simplify a fraction: Example: How do we simplify 1230 ? Earlier we found that the Common Factors of 12 and 30 are 1, 2, 3 and 6, and so the Greatest Common Factor is 6. So the largest number we can divide both 12 and 30 exactly by is 6, like this: ÷ 6 1230 = 25 ÷ 6 The Greatest Common Factor of 12 and 30 is 6. And so 1230 can be simplified to 25 Finding the Greatest Common Factor Here are three ways: 1. We can: find all factors of both numbers (use the All Factors Calculator), then find the ones that are common to both, and then choose the greatest Example: Two Numbers Factors Common Factors Greatest Common Factor Example Simplified Fraction 9 and 12 9: 1, 3, 9 12: 1, 2, 3, 4, 6, 12 1, 3 3 912 = 34 And another example: Two Numbers Factors Common Factors Greatest Common Factor Example Simplified Fraction 6 and 18 6: 1, 2, 3, 6 18: 1, 2, 3, 6, 9, 18 1, 2, 3, 6 6 618 = 13 2. Or we can find the prime factors and combine the common ones together: Two Numbers Thinking ... Greatest Common Factor Example Simplified Fraction 24 and 108 2 × 2 × 2 × 3 = 24, and 2 × 2 × 3 × 3 × 3 = 108 2 × 2 × 3 = 12 24108 = 29 3. Or sometimes we can just play around with the factors until we discover it: Two Numbers Thinking ... Greatest Common Factor Example Simplified Fraction 9 and 12 3 × 3 = 9 and 3 × 4 = 12 3 912 = 34 But in that case we must check that we have found the greatest common factor. Greatest Common Factor Calculator OK, there is also a really easy method: we can use the Greatest Common Factor Calculator to find it automatically. Other Names The "Greatest Common Factor" is often abbreviated to GCF, and is also known as: the "Greatest Common Divisor" or GCD the "Highest Common Factor" or HCF 920,921,1382,1383,3548,3549,3550,3551,5010,5011 Copyright © 2025 Rod Pierce Factoring is the opposite of distributing. When distributing, you multiply a series of terms by a common factor. When factoring, you seek to find what a series of terms have in common and then take it away, dividing the common factor out from each term. Think of each term as a numerator and then find the same denominator for each. By factoring out, the factor is put outside the parentheses or brackets and all the results of the divisions are left inside. The proper way to factor an expression is to write the prime factorization of each of the numbers and look for the greatest common factor. A more practical and quicker way is to look for the largest factor that you can easily recognize. Factor it out and then see if the numbers within the parentheses need to be factored again. Repeat the division until the terms within the parentheses are relatively prime. Example: Follow these steps to factor out the expression Determine a common factor. A common factor is 2. Divide each term by the common factor and write the results of the division in parentheses, with the factor out in front. Determine whether you can factor out any other terms. The terms left in the parentheses are still too large. They all still a common factor of 4. Factoring out 4, you get: Simplify the answer. If you factor out a 4 after factoring out the 2, then the product of 4 and 2 (which is 8), is the total amount you factored out. The final answer is