Newton’s Laws of Motion form the cornerstone of classical mechanics, a fundamental branch of physics that deals with the behavior of objects in motion. These laws, formulated by Sir Isaac Newton in the 17th century, have profound implications in the field of engineering, particularly in dynamics and control. Understanding these laws is crucial for engineers as they provide the foundational principles for analyzing and designing systems that involve motion. From the simplest mechanical devices to complex aerospace systems, Newton’s Laws of Motion are indispensable tools that enable engineers to predict and control the behavior of physical systems. Fundamentals Newton’s First Law of Motion Also known as the Law of Inertia, Newton’s First Law states that an object will remain at rest or in uniform motion in a straight line unless acted upon by an external force. This principle highlights the concept of inertia, which is the tendency of an object to resist changes in its state of motion. Newton’s Second Law of Motion Newton’s Second Law provides a quantitative description of the changes that a force can produce on the motion of an object. It states that the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass. Mathematically, it is expressed as: F = ma where F is the net force, m is the mass, and a is the acceleration. Newton’s Third Law of Motion Newton’s Third Law states that for every action, there is an equal and opposite reaction. This means that forces always occur in pairs; if one object exerts a force on another, the second object exerts an equal and opposite force on the first. Key Terms Inertia: The resistance of an object to any change in its state of motion. Force: An interaction that causes an object to change its velocity, direction, or shape. Mass: A measure of the amount of matter in an object, which also determines its resistance to acceleration. Acceleration: The rate of change of velocity of an object. Historical Development The development of Newton’s Laws of Motion marked a significant milestone in the history of science and engineering. Before Newton, the understanding of motion was largely based on the works of Aristotle, who believed that a force was necessary to maintain motion. This view was challenged by Galileo Galilei in the early 17th century, who conducted experiments that demonstrated the concept of inertia. Sir Isaac Newton built upon Galileo’s work and formulated his three laws of motion, which he published in his seminal work, Philosophiæ Naturalis Principia Mathematica, in 1687. This work not only laid the foundation for classical mechanics but also revolutionized the way scientists and engineers understood the physical world. Newton’s contributions have had a lasting impact, influencing countless advancements in various fields of science and engineering. Applications Newton’s Laws of Motion have a wide range of practical applications across various industries and fields. Here are some notable examples: Automotive Engineering In automotive engineering, Newton’s Second Law is used to design and analyze the performance of vehicles. Engineers use this law to calculate the forces required for acceleration, braking, and cornering. For instance, understanding the relationship between force, mass, and acceleration helps in optimizing engine performance and fuel efficiency. Aerospace Engineering Newton’s Laws are fundamental in aerospace engineering for designing and controlling aircraft and spacecraft. The principles of motion are used to calculate the thrust required for takeoff, the forces acting on an aircraft during flight, and the trajectories of spacecraft. Newton’s Third Law is particularly important in rocketry, where the expulsion of exhaust gases generates the thrust needed to propel the rocket. Robotics In robotics, Newton’s Laws are applied to design and control robotic systems. Engineers use these principles to model the dynamics of robots, predict their motion, and develop control algorithms. For example, understanding the forces and torques acting on a robotic arm is essential for precise manipulation and movement. Structural Engineering Newton’s Laws are also crucial in structural engineering for analyzing the forces acting on buildings, bridges, and other structures. Engineers use these principles to ensure that structures can withstand various loads and forces, such as wind, earthquakes, and traffic. Advanced Topics Nonlinear Dynamics While Newton’s Laws provide a solid foundation for understanding motion, many real-world systems exhibit nonlinear behavior that requires more advanced analysis. Nonlinear dynamics involves the study of systems where the relationship between variables is not proportional. This field has applications in areas such as chaos theory, fluid dynamics, and control systems. Control Theory Control theory is an advanced field that deals with the behavior of dynamical systems and the design of controllers to achieve desired performance. Newton’s Laws are integral to developing mathematical models of systems, which are then used to design control strategies. Recent research in control theory includes adaptive control, robust control, and optimal control, which aim to improve the performance and stability of complex systems. Computational Mechanics With the advent of powerful computers, computational mechanics has become an essential tool for solving complex problems in dynamics and control. Numerical methods and simulations based on Newton’s Laws are used to analyze and predict the behavior of systems that are difficult to solve analytically. This approach is widely used in fields such as finite element analysis, computational fluid dynamics, and multibody dynamics. Despite their wide applicability, Newton’s Laws of Motion have certain limitations and challenges: Relativistic Effects Newton’s Laws are based on classical mechanics and are not accurate at very high speeds, close to the speed of light. In such cases, relativistic mechanics, as described by Einstein’s theory of relativity, must be used. Quantum Effects At the microscopic scale, Newton’s Laws do not accurately describe the behavior of particles. Quantum mechanics provides a more accurate framework for understanding the motion and interactions of subatomic particles. Complex Systems Many real-world systems are highly complex and involve multiple interacting components. Analyzing and controlling such systems can be challenging due to the nonlinear and dynamic nature of their behavior. Advanced techniques in systems engineering and control theory are often required to address these challenges. Environmental Factors External factors such as friction, air resistance, and temperature can affect the motion of objects and must be considered in practical applications. Engineers often need to account for these factors to ensure accurate predictions and reliable performance. Conclusion Newton’s Laws of Motion are fundamental principles that underpin the field of dynamics and control in engineering. These laws provide the essential tools for analyzing and predicting the behavior of physical systems, enabling engineers to design and control a wide range of applications, from vehicles and aircraft to robots and structures. While there are challenges and limitations associated with these laws, ongoing research and advancements in related fields continue to expand our understanding and capabilities. The importance of Newton’s Laws in engineering cannot be overstated, as they remain a cornerstone of classical mechanics and a vital component of modern engineering practice. Newton’s Second Law of Motion is a fundamental principle that describes the relationship between force, mass, and acceleration. This guide breaks down the law into easily understandable terms, complemented by practical examples. Ideal for educational purposes, it explains how this law is pivotal in understanding the mechanics of motion. From everyday occurrences to complex scientific phenomena, this guide illustrates the law’s applications, making it a vital resource for teachers and students delving into physics. What is Newton’s Second Law of Motion – Definition Newton’s Second Law of Motion states that the force acting on an object is equal to the mass of that object multiplied by its acceleration. This law quantifies the concept of force and provides a formula for calculating how forces affect the motion of objects. It is a key concept in physics, offering a mathematical framework for understanding how and why objects move the way they do. What is the Best Example of Newton’s Second Law of Motion? A classic example of Newton’s Second Law is a car accelerating on a road. The force applied by the car’s engine produces an acceleration that is directly proportional to the force and inversely proportional to the mass of the car. This example demonstrates how varying the force or the mass can affect the acceleration of an object. Newton’s Second Law of Motion Formula This formula is a fundamental equation in physics, used to calculate how much force is needed to move an object at a certain acceleration, or conversely, to determine the acceleration that a given force will produce on an object of a certain mass. F = ma Where: “F” represents the force applied to an object. “m” stands for the mass of the object. “a” denotes the acceleration of the object. Newton’s Second Law of Motion Unit Newton’s Second Law of Motion, represented by the equation F = ma (Force equals mass times acceleration), establishes a foundational relationship in physics, linking force, mass, and acceleration. The standard unit of force, as derived from this law, is the Newton (N), which is a compound unit composed of the units for mass and acceleration. Specifically, one Newton is equivalent to the force required to accelerate one kilogram of mass at the rate of one meter per second squared(1 N = 1 $kg \cdot m/s^2$). 22 Newton’s Second Law of Motion Examples Newton’s Second Law of Motion, encapsulating the relationship between force, mass, and acceleration, is a pivotal concept in understanding the dynamics of motion. This collection of 22 examples illuminates the law’s application in various contexts, providing a comprehensive perspective for educators. Each example highlights how changes in force and mass influence acceleration, demonstrating the law’s practical relevance. From everyday occurrences to complex technological systems, these instances serve as invaluable teaching aids, enhancing students’ grasp of this fundamental physical principle. Pushing a Shopping Cart: More force is required to accelerate a full cart than an empty one. Kicking a Soccer Ball: The harder the kick (force), the faster the ball accelerates. Driving a Car: Acceleration increases with more engine force; heavier cars need more force. Shooting a Basketball: Varying the shooting force changes the ball’s acceleration. Launching a Rocket: Tremendous force is required to accelerate the massive rocket. Swinging a Golf Club: The club’s force determines the ball’s acceleration and distance. Pulling a Wagon: More force is needed to accelerate a wagon with passengers. Bicycling Uphill: Requires more force compared to level ground for the same acceleration. Skydiving: The force of gravity accelerates the diver until air resistance balances it. Using a Sling Shot: Stretching it more (applying more force) accelerates the projectile faster. Hitting a Baseball: The bat’s force affects the ball’s speed and trajectory. Rowing a Boat: Force applied on the oars determines the boat’s acceleration. Jumping from a Height: The force upon landing is greater for heavier individuals. Elevator Movement: More force is needed for faster acceleration or heavier loads. Throwing a Dart: The acceleration of the dart depends on the throwing force. Skating on Ice: Pushing harder against the ice results in faster acceleration. Starting a Lawnmower: The pull force affects the starter cord’s acceleration. Accelerating a Motorcycle: More throttle applies greater force, increasing acceleration. Bowling: The force imparted determines the ball’s speed. Snowboarding Downhill: Gravitational force accelerates the snowboarder. A Hammer Striking a Nail: The hammer’s force determines the nail’s acceleration into wood. A Catapult Launching: The tension force dictates the acceleration and distance of the projectile. Newton’s Second Law of Motion Examples In Everyday Life Newton’s Second Law of Motion, illustrating the relationship between force, mass, and acceleration, is evident in many common activities. These examples shed light on the law’s application in everyday scenarios, enhancing its comprehension. By observing these instances, students can see how varying the force applied or the mass of an object influences its acceleration, making the concept more tangible and relatable in daily life. Examples: Pushing a Grocery Cart: Heavier carts require more force to accelerate to the same speed as lighter ones. Using a Hammer: The force applied to the hammer influences the speed it drives a nail into wood. Stepping on a Gas Pedal: The harder you press, the more force is applied, accelerating the car faster. Sliding Furniture Across the Floor: Heavier furniture needs more force to achieve the same acceleration as lighter pieces. Using a Blender: Higher speed settings apply more force, causing the blades to accelerate faster. Newton’s Second Law of Motion Examples In Real Life Newton’s Second Law is not just a theoretical concept; it’s actively at work in various real-life situations. These examples help illustrate how the law operates in practical, everyday contexts, offering clear insights into the dynamics of motion and force. Understanding these applications aids in connecting theoretical physics to real-world experiences. Examples: Lifting Weights: Heavier weights require more force to lift at the same speed as lighter ones. Accelerating Bicycles: More force is needed to accelerate a bike with a rider than without. Braking a Vehicle: Heavier vehicles require more force to decelerate to a stop than lighter ones. Watering Plants with a Hose: Increasing the water pressure (force) accelerates the water out of the hose. Climbing Stairs: More effort (force) is needed to ascend faster. Newton’s Second Law of Motion Examples In Daily Life The principles of Newton’s Second Law of Motion are a constant presence in our daily lives, guiding the motion of objects we interact with. These examples demonstrate the law’s relevance in day-to-day activities, offering an intuitive understanding of force, mass, and acceleration. Examples: Pushing a Child on a Swing: Applying more force results in higher and faster swings. Vacuuming a Rug: More force is required to move the vacuum cleaner over a thicker rug. Squeezing a Ketchup Bottle: The amount of force applied determines the speed of ketchup coming out. Opening and Closing Doors: Heavier doors require more force to open and close at the same speed. Throwing a Ball: The force behind the throw affects how fast and far the ball travels. Newton’s Second Law of Motion Examples In Sports In sports, Newton’s Second Law of Motion plays a crucial role in understanding how athletes and objects move. These examples show how the law applies in various sports, providing insights into how athletes can optimize their performance by understanding the relationship between force, mass, and acceleration. Examples: Baseball Pitching: The force exerted by the pitcher affects the ball’s acceleration and speed. Soccer Kicks: Kicking the ball with more force results in a faster and longer shot. Gymnastics Vault: The gymnast’s force against the vaulting table determines their acceleration in the air. Bowling: The force applied to the bowling ball influences its speed and the power of the strike. Golf Swings: The acceleration and distance of the golf ball are determined by the force of the swing. Importance of Newton’s Second Law of Motion Newton’s Second Law of Motion is a fundamental principle in physics, providing a quantitative description of the dynamics of force and motion. This law is critical for understanding how forces affect the motion of objects. It bridges the gap between theoretical physics and practical applications, from designing vehicles to understanding natural phenomena. The law’s universality and applicability make it a cornerstone in the study of mechanics and a crucial tool in various scientific and engineering fields. Importance Points: Mechanics Foundation: Fundamental in classical mechanics for understanding motion. Engineering Applications: Essential in designing mechanical systems and structures. Space Exploration: Crucial for calculating spacecraft trajectories and propulsion. Safety Mechanisms: Helps in designing vehicle safety features like airbags. Sports Science: Assists in optimizing athletes’ performance. Physics Education: Key concept in teaching dynamics and forces. Technological Innovations: Underpins the development of various technologies. Understanding Nature: Explains natural occurrences like tidal movements. Industrial Machinery: Guides the design and operation of machinery. Medical Equipment Design: Important in creating devices like prosthetics. Application of Newton’s Second Law of Motion The application of Newton’s Second Law of Motion spans multiple disciplines, offering a practical framework for understanding and manipulating forces and motion. This law serves as a guide in various fields for predicting the behavior of objects under force, aiding in problem-solving and innovation. Application Steps: Identify the Object: Determine the object of interest in a motion scenario. Measure the Mass: Ascertain the mass of the object. Determine the Force: Calculate or measure the force applied to the object. Calculate Acceleration: Use the law (F=ma) to compute acceleration. Predict Motion: Predict the object’s motion based on calculated acceleration. Design and Test: Apply the law in designing systems and validate with experiments. Optimize Performance: Use the law to improve efficiency and effectiveness in systems. Why is Newton’s Second Law called a law of momentum? Newton’s Second Law of Motion is often referred to as the law of momentum because it describes the relationship between force and the change in momentum. Momentum, defined as the product of mass and velocity (p=mv), is a key concept in physics. The law states that the force applied to an object is equal to the rate of change of its momentum (F=dt/dp). This highlights how force influences an object’s momentum, making it a fundamental law in understanding and analyzing motion and forces. How do you verify Newton’s Second Law of Motion? Verifying Newton’s Second Law of Motion involves a series of steps to experimentally demonstrate the relationship between force, mass, and acceleration. These experiments typically measure how different forces affect the acceleration of objects with varying masses. Verification Steps: Set Up Experiment: Use a dynamic cart, a track, and a pulley system. Measure Mass: Determine the mass of the cart. Apply Force: Use hanging weights to apply a known force to the cart. Record Acceleration: Measure the acceleration of the cart using sensors or timers. Repeat with Variations: Change the mass of the cart or the force applied. Analyze Results: Compare the measured accelerations against the predicted values from F=ma. Confirm Relationship: Verify that the acceleration is directly proportional to the force and inversely proportional to the mass. How to Use Newton’s Second Law to Calculate Acceleration The formula for Newton’s second law or the law of acceleration is a= F/m, Where a is the amount of acceleration (m/s^2 or meters per second squared), F is the total amount of force or net force (N or Newtons), and m is the total mass of the object (kg). Step 1: Write Down the Formula for Acceleration Begin by writing down the formula of acceleration on your worksheet or a digital note-taking software of your choice. This will help outline the steps you will have to do and will provide structure for your final output. Step 2: List Out the Given Variables After you have written down the formula for acceleration, you must list out the given variables the question has provided for you. Listing them out will prevent you from getting confused with the other variables in the question. Step 3: Change and Ensure the Variables are Using the Correct Measurements Ensure that each of the variables you have listed is using the correct measurements used in the formula. This means that if there is a discrepancy between the measurement of a specific variable and the variable used in the formula, you will need to convert a said variable into the correct measurement. Step 4: Create the Equation via Substitution When you have ensured that the variables are in the correct measurement, you can now substitute the variables into the solution. Doing the substitution will create a working equation where you may find the missing variable. Note that the missing variable has to be on the left side of the acceleration equation, which means you must maneuver all the variables to the correct positions. Step 5: Answer the Equation by doing the correct mathematical operation. Note that the measurement provided by the equation is static, which means that if the question is asking for a specific measurement you will need to convert the answer to the correct form of measurement. FAQs What is Newton’s first law called? Newton’s first law of motion is dubbed the law of inertia. This law states that if an object or a body is at rest it will continue to be at rest unless acted upon by an unbalanced and external force. An example of this law in action is seen in the movement of chairs. The chair will stay in place if no external force is applied to it and will be in a state of rest. But if an outside force is applied to the chair that is greater than the inertia exerted by the said chair, then it will move to a specific distance and direction based on the force acting upon it. Following this law, if the chair’s inertia is stronger than the external force, then it will not move from its position. What is Newton’s third law called? Newton’s third law of motion is called the law of action and reaction. This law indicates that every force or action found in nature has an equal and opposite reaction. One can easily observe this law in the tugging motion of the game tug-of-war. Wherein one side pulls on the rope with a specific amount of force, while the other tries to pull it on their side with the same or more amount of force in the opposite direction. This will cause a cycle of actions and reactions in the form of pulling and resistance, which will end when one side overcomes the other. If one were to follow this law, a specific action creates a cause while the reaction is the effect of said cause. What is inertia and how does it relate to Newton’s second law? Inertia is a specific amount of force internally exerted by an object that will try and resist an external force that will change its position or direction. This force affects all of Newton’s laws of motion as it will be the driving force of an object’s resistance to movement or change in direction. Both inertia and gravity affect the acceleration of a specific object accelerating in a single direction, as it will try and resist the movement brought about by acceleration. Newton’s second law of motion or the law of acceleration allows people to understand how things in this world move and pick up speed as it trails in a single direction. Newton’s second law in action has allowed people to manufacture transportation that will allow other people to traverse large amounts of distances with greater ease and accessibility. Therefore it is important to understand the law of acceleration and how it affects a lot of things in our everyday life. Laws in physics about force and motion "Newton's laws" redirects here. For other uses, see Newton's law. "F=ma" redirects here. For the physics competition, see F=ma exam. Part of a series onClassical mechanics F = d p d t {\displaystyle {\textbf {F}}={\frac {d\mathbf {p} }{dt}}} Second law of motion History Timeline Textbooks Branches Applied Celestial Continuum Dynamics Field theory Kinematics Kinetics Statics Statistical mechanics Fundamentals Acceleration Angular momentum Couple D'Alembert's principle Energy kinetic potential Force Frame of reference Inertial frame of reference Impulse Inertia / Moment of inertia Mass Mechanical power Mechanical work Moment Momentum Space Speed Time Torque Velocity Virtual work Formulations Newton's laws of motion Analytical mechanics Lagrangian mechanicsHamiltonian mechanicsRouthian mechanicsHamilton–Jacobi equationAppell's equation of motionKoopman–von Neumann mechanics Core topics Damping Displacement Equations of motion Euler's laws of motion Fictitious force Friction Harmonic oscillator Inertial / Non-inertial reference frame Motion (linear) Newton's law of universal gravitation Newton's laws of motion Relative velocity Rigid body dynamics Euler's equations Simple harmonic motion Vibration Rotation Circular motion Rotating reference frame Centripetal force Centrifugal force reactive Coriolis force Pendulum Tangential speed Rotational frequency Angular acceleration / displacement / frequency / velocity Scientists Kepler Galileo Huygens Newton Horrocks Halley Maupertuis Daniel Bernoulli Johann Bernoulli Euler d'Alembert Clairaut Lagrange Laplace Poisson Hamilton Jacobi Cauchy Routh Liouville Appell Gibbs Koopman von Neumann Physics portal Categoryvte Newton's laws of motion are three physical laws that describe the relationship between the motion of an object and the forces acting on it. These laws, which provide the basis for Newtonian mechanics, can be paraphrased as follows: A body remains at rest, or in motion at a constant speed in a straight line, unless it is acted upon by a force. At any instant of time, the net force on a body is equal to the body's acceleration multiplied by its mass or, equivalently, the rate at which the body's momentum is changing with time. If two bodies exert forces on each other, these forces have the same magnitude but opposite directions.[1][2] The three laws of motion were first stated by Isaac Newton in his Philosophiæ Naturalis Principia Mathematica (Mathematical Principles of Natural Philosophy), originally published in 1687.[3] Newton used them to investigate and explain the motion of many physical objects and systems. In the time since Newton, new insights, especially around the concept of energy, built the field of classical mechanics on his foundations. Limitations to Newton's laws have also been discovered; new theories are necessary when objects move at very high speeds (special relativity), are very massive (general relativity), or are very small (quantum mechanics). Newton's laws are often stated in terms of point or particle masses, that is, bodies whose volume is negligible. This is a reasonable approximation for real bodies when the motion of internal parts can be neglected, and when the separation between bodies is much larger than the size of each. For instance, the Earth and the Sun can both be approximated as pointlike when considering the orbit of the former around the latter, but the Earth is not pointlike when considering activities on its surface.[note 1] The mathematical description of motion, or kinematics, is based on the idea of specifying positions using numerical coordinates. Movement is represented by these numbers changing over time: a body's trajectory is represented by a function that assigns to each value of a time variable the values of all the position coordinates. The simplest case is one-dimensional, that is, when a body is constrained to move only along a straight line. Its position can then be given by a single number, indicating where it is relative to some chosen reference point. For example, a body might be free to slide along a track that runs left to right, and so its location can be specified by its distance from a convenient zero point, or origin, with negative numbers indicating positions to the left and positive numbers indicating positions to the right. If the body's location as a function of time is s (t) {\displaystyle s(t)} , then its average velocity over the time interval from t 0 {\displaystyle t_{0}} to t 1 {\displaystyle t_{1}} is[6] Δ s Δ t = s (t 1) − s (t 0) t 1 − t 0 . {\displaystyle {\frac {\Delta s}{\Delta t}}={\frac {s(t_{1})-s(t_{0})}{t_{1}-t_{0}}}.} Here, the Greek letter Δ {\displaystyle \Delta } (delta) is used, per tradition, to mean "change in". A positive average velocity means that the position coordinate s {\displaystyle s} increases over the interval in question, a negative average velocity indicates a net decrease over that interval, and an average velocity of zero means that the body ends the time interval in the same place as it began. Calculus gives the means to define an instantaneous velocity, a measure of a body's speed and direction of movement at a single moment of time, rather than over an interval. One notation for the instantaneous velocity is to replace Δ {\displaystyle \Delta } with the symbol d {\displaystyle d} , for example, v = d s d t . {\displaystyle v={\frac {ds}{dt}}.} This denotes that the instantaneous velocity is the derivative of the position with respect to time. It can roughly be thought of as the ratio between an infinitesimally small change in position d s {\displaystyle ds} to the infinitesimally small time interval d t {\displaystyle dt} over which it occurs.[7] More carefully, the velocity and all other derivatives can be defined using the concept of a limit.[6] A function f (t) {\displaystyle f(t)} has a limit of L {\displaystyle L} at a given input value t 0 {\displaystyle t_{0}} if the difference between f {\displaystyle f} and L {\displaystyle L} can be made arbitrarily small by choosing an input sufficiently close to t 0 {\displaystyle t_{0}} . One writes, lim t → t 0 f (t) = L . {\displaystyle \lim _{t\to t_{0}}f(t)=L.} Instantaneous velocity can be defined as the limit of the average velocity as the time interval shrinks to zero: d s d t = lim Δ t → 0 s (t + Δ t) − s (t) Δ t . {\displaystyle {\frac {ds}{dt}}=\lim _{\Delta t\to 0}{\frac {s(t+\Delta t)-s(t)}{\Delta t}}.} Acceleration is to velocity as velocity is to position: it is the derivative of the velocity with respect to time.[note 2] Acceleration can likewise be defined as a limit: a = d v d t = lim Δ t → 0 v (t + Δ t) − v (t) Δ t . {\displaystyle a={\frac {dv}{dt}}=\lim _{\Delta t\to 0}{\frac {v(t+\Delta t)-v(t)}{\Delta t}}.} Consequently, the acceleration is the second derivative of position,[7] often written d 2 s d t 2 {\displaystyle {\frac {d^{2}s}{dt^{2}}}} . Position, when thought of as a displacement from an origin point, is a vector: a quantity with both magnitude and direction.[9]:1 Velocity and acceleration are vector quantities as well. The mathematical tools of vector algebra provide the means to describe motion in two, three or more dimensions. Vectors are often denoted with an arrow, as in s → {\displaystyle {\vec {s}}} , or in bold typeface, such as s {\displaystyle {\bf {s}}} . Often, vectors are represented visually as arrows, with the direction of the vector being the direction of the arrow, and the magnitude of the vector indicated by the length of the arrow. Numerically, a vector can be represented as a list; for example, a body's velocity vector might be v = (3   m / s , 4   m / s) {\displaystyle \mathbf {v} =(\mathrm {3~m/s} ,\mathrm {4~m/s})} , indicating that it is moving at 3 metres per second along the horizontal axis and 4 metres per second along the vertical axis. The same motion described in a different coordinate system will be represented by different numbers, and vector algebra can be used to translate between these alternatives.[9]:4 The study of mechanics is complicated by the fact that household words like energy are used with a technical meaning.[10][11] Moreover, words which are synonymous in everyday speech are not so in physics: force is not the same as power or pressure, for example, and mass has a different meaning than weight.[12][13]:150 The physics concept of force makes quantitative the everyday idea of a push or a pull. Forces in Newtonian mechanics are often due to strings and ropes, friction, muscle effort, gravity, and so forth. Like displacement, velocity, and acceleration, force is a vector quantity. Artificial satellites move along curved orbits, rather than in straight lines, because of the Earth's gravity. Translated from Latin, Newton's first law reads, Every object perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon.[note 3] Newton's first law expresses the principle of inertia: the natural behavior of a body is to move in a straight line at constant speed. A body's motion preserves the status quo, but external forces can perturb this. The modern understanding of Newton's first law is that no inertial observer is privileged over any other. The concept of an inertial observer makes quantitative the everyday idea of feeling no effects of motion. For example, a person standing on the ground watching a train go past is an inertial observer. If the observer on the ground sees the train moving smoothly in a straight line at a constant speed, then a passenger sitting on the train will also be an inertial observer: the train passenger feels no motion. The principle expressed by Newton's first law is that there is no way to say which inertial observer is "really" moving and which is "really" standing still. One observer's state of rest is another observer's state of uniform motion in a straight line, and no experiment can deem either point of view to be correct or incorrect. There is no absolute standard of rest.[18][15]:62–63[19]:7–9 Newton himself believed that absolute space and time existed, but that the only measures of space or time accessible to experiment are relative.[20] The change of motion of an object is proportional to the force impressed; and is made in the direction of the straight line in which the force is impressed.[15]:114 By "motion", Newton meant the quantity now called momentum, which depends upon the amount of matter contained in a body, the speed at which that body is moving, and the direction in which it is moving.[21] In modern notation, the momentum of a body is the product of its mass and its velocity: p = m v , {\displaystyle \mathbf {p} =m\mathbf {v} \,,} where all three quantities can change over time. In common cases the mass m {\displaystyle m} does not change with time and the derivative acts only upon the velocity. Then force equals the product of the mass and the time derivative of the velocity, which is the acceleration:[22] F = m d v d t = m a . {\displaystyle \mathbf {F} =m{\frac {d\mathbf {v} }{dt}}=m\mathbf {a} \,.} As the acceleration is the second derivative of position with respect to time, this can also be written F = m d 2 s d t 2 . {\displaystyle \mathbf {F} =m{\frac {d^{2}\mathbf {s} }{dt^{2}}}.} Newton's second law, in modern form, states that the time derivative of the momentum is the force:[23]:4.1 F = d p d t . {\displaystyle \mathbf {F} ={\frac {d\mathbf {p} }{dt}}\,.} When applied to systems of variable mass, the equation above is only valid only for a fixed set of particles. Applying the derivative as in F = m d v d t + v d m d t   (i n c o r r e c t) {\displaystyle \mathbf {F} =m{\frac {\mathrm {d} \mathbf {v} }{\mathrm {d} t}}+\mathbf {v} {\frac {\mathrm {d} m}{\mathrm {d} t}}\ \ \mathrm {(incorrect)} } can lead to incorrect results.[24] For example, the momentum of a water jet system must include the momentum of the ejected water:[25] F e x t = d p d t − v e j e c t d m d t . {\displaystyle \mathbf {F} _{\mathrm {ext} }={\mathrm {d} \mathbf {p} \over \mathrm {d} t}-\mathbf {v} _{\mathrm {eject} }{\frac {\mathrm {d} m}{\mathrm {d} t}}.} A free body diagram for a block on an inclined plane, illustrating the normal force perpendicular to the plane (N), the downward force of gravity (mg), and a force f along the direction of the plane that could be applied, for example, by friction or a string The forces acting on a body add as vectors, and so the total force on a body depends upon both the magnitudes and the directions of the individual forces.[23]:58 When the net force on a body is equal to zero, then by Newton's second law, the body does not accelerate, and it is said to be in mechanical equilibrium. A state of mechanical equilibrium is stable if, when the position of the body is changed slightly, the body remains near that equilibrium. Otherwise, the equilibrium is unstable.[15]:121[23]:174 A common visual representation of forces acting in concert is the free body diagram, which schematically portrays a body of interest and the forces applied to it by outside influences.[26] For example, a free body diagram of a block sitting upon an inclined plane can illustrate the combination of gravitational force, "normal" force, friction, and string tension.[note 4] Newton's second law is sometimes presented as a definition of force, i.e., a force is that which exists when an inertial observer sees a body accelerating. This is sometimes regarded as a potential tautology — acceleration implies force, force implies acceleration. However, Newton's second law not only merely defines the force by the acceleration: forces exist as separate from the acceleration produced by the force in a particular system. The same force that is identified as producing acceleration to an object can then be applied to any other object, and the resulting accelerations (coming from that same force) will always be inversely proportional to the mass of the object. What Newton's Second Law states is that all the effect of a force onto a system can be reduced to two pieces of information: the magnitude of the force, and it's direction, and then goes on to specify what the effect is. Beyond that, an equation detailing the force might also be specified, like Newton's law of universal gravitation. By inserting such an expression for F {\displaystyle \mathbf {F} } into Newton's second law, an equation with predictive power can be written.[note 5] Newton's second law has also been regarded as setting out a research program for physics, establishing that important goals of the subject are to identify the forces present in nature and to catalogue the constituents of matter.[15]:134[28]:12-2 However, forces can often be measured directly with no acceleration being involved, such as through weighing scales. By postulating a physical object that can be directly measured independently from acceleration, Newton made a objective physical statement with the second law alone, the predictions of which can be verified even if no force law is given. To every action, there is always opposed an equal reaction; or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.[15]:116 Rockets work by creating unbalanced high pressure that pushes the rocket upwards while exhaust gas exits through an open nozzle.[30] In other words, if one body exerts a force on a second body, the second body is also exerting a force on the first body, of equal magnitude in the opposite direction. Overly brief paraphrases of the third law, like "action equals reaction" might have caused confusion among generations of students: the "action" and "reaction" apply to different bodies. For example, consider a book at rest on a table. The Earth's gravity pulls down upon the book. The "reaction" to that "action" is not the support force from the table holding up the book, but the gravitational pull of the book acting on the Earth.[note 6] Newton's third law relates to a more fundamental principle, the conservation of momentum. The latter remains true even in cases where Newton's statement does not, for instance when force fields as well as material bodies carry momentum, and when momentum is defined properly, in quantum mechanics as well.[note 7] In Newtonian mechanics, if two bodies have momenta p 1 {\displaystyle \mathbf {p} _{1}} and p 2 {\displaystyle \mathbf {p} _{2}} respectively, then the total momentum of the pair is p = p 1 + p 2 {\displaystyle \mathbf {p} =\mathbf {p} _{1}+\mathbf {p} _{2}} , and the rate of change of p {\displaystyle \mathbf {p} } is d p d t = d p 1 d t + d p 2 d t . {\displaystyle {\frac {d\mathbf {p} }{dt}}={\frac {d\mathbf {p} _{1}}{dt}}+{\frac {d\mathbf {p} _{2}}{dt}}.} By Newton's second law, the first term is the total force upon the first body, and the second term is the total force upon the second body. If the two bodies are isolated from outside influences, the only force upon the first body can be that from the second, and vice versa. By Newton's third law, these forces have equal magnitude but opposite direction, so they cancel when added, and p {\displaystyle \mathbf {p} } is constant. Alternatively, if p {\displaystyle \mathbf {p} } is known to be constant, it follows that the forces have equal magnitude and opposite direction. Various sources have proposed elevating other ideas used in classical mechanics to the status of Newton's laws. For example, in Newtonian mechanics, the total mass of a body made by bringing together two smaller bodies is the sum of their individual masses. Frank Wilczek has suggested calling attention to this assumption by designating it "Newton's Zeroth Law".[37] Another candidate for a "zeroth law" is the fact that at any instant, a body reacts to the forces applied to it at that instant.[38] Likewise, the idea that forces add like vectors (or in other words obey the superposition principle), and the idea that forces change the energy of a body, have both been described as a "fourth law".[note 8] Moreover, some texts organize the basic ideas of Newtonian mechanics into different postulates, other than the three laws as commonly phrased, with the goal of being more clear about what is empirically observed and what is true by definition.[19]:9[27] The study of the behavior of massive bodies using Newton's laws is known as Newtonian mechanics. Some example problems in Newtonian mechanics are particularly noteworthy for conceptual or historical reasons. Main articles: Free fall and Projectile motion A bouncing ball photographed at 25 frames per second using a stroboscopic flash. In between bounces, the ball's height as a function of time is close to being a parabola, deviating from a parabolic arc because of air resistance, spin, and deformation into a non-spherical shape upon impact. If a body falls from rest near the surface of the Earth, then in the absence of air resistance, it will accelerate at a constant rate. This is known as free fall. The speed attained during free fall is proportional to the elapsed time, and the distance traveled is proportional to the square of the elapsed time.[43] Importantly, the acceleration is the same for all bodies, independently of their mass. This follows from combining Newton's second law of motion with his law of universal gravitation. The latter states that the magnitude of the gravitational force from the Earth upon the body is F = G M m r 2 , {\displaystyle F={\frac {GMm}{r^{2}}},} where m {\displaystyle m} is the mass of the falling body, M {\displaystyle M} is the mass of the Earth, G {\displaystyle G} is Newton's constant, and r {\displaystyle r} is the distance from the center of the Earth to the body's location, which is very nearly the radius of the Earth. Setting this equal to m a {\displaystyle ma} , the body's mass m {\displaystyle m} cancels from both sides of the equation, leaving an acceleration that depends upon G {\displaystyle G} , M {\displaystyle M} , and r {\displaystyle r} , and r {\displaystyle r} can be taken to be constant. This particular value of acceleration is typically denoted g {\displaystyle g} : g = G M r 2 ≈ 9.8   m / s 2 . {\displaystyle g={\frac {GM}{r^{2}}}\approx \mathrm {9.8~m/s^{2}} .} If the body is not released from rest but instead launched upwards and/or horizontally with nonzero velocity, then free fall becomes projectile motion.[44] When air resistance can be neglected, projectiles follow parabola-shaped trajectories, because gravity affects the body's vertical motion and not its horizontal. At the peak of the projectile's trajectory, its vertical velocity is zero, but its acceleration is g {\displaystyle g} downwards, as it is at all times. Setting the wrong vector equal to zero is a common confusion among physics students.[45] Main article: Circular motion Two objects in uniform circular motion, orbiting around the barycenter (center of mass of both objects) When a body is in uniform circular motion, the force on it changes the direction of its motion but not its speed. For a body moving in a circle of radius r {\displaystyle r} at a constant speed v {\displaystyle v} , its acceleration has a magnitude a = v 2 r {\displaystyle a={\frac {v^{2}}{r}}} and is directed toward the center of the circle.[note 9] The force required to sustain this acceleration, called the centripetal force, is therefore also directed toward the center of the circle and has magnitude m v 2 / r {\displaystyle mv^{2}/r} . Many orbits, such as that of the Moon around the Earth, can be approximated by uniform circular motion. In such cases, the centripetal force is gravity, and by Newton's law of universal gravitation has magnitude G M m / r 2 {\displaystyle GMm/r^{2}} , where M {\displaystyle M} is the mass of the larger body being orbited. Therefore, the mass of a body can be calculated from observations of another body orbiting around it.[47]:130 Newton's cannonball is a thought experiment that interpolates between projectile motion and uniform circular motion. A cannonball that is lobbed weakly off the edge of a tall cliff will hit the ground in the same amount of time as if it were dropped from rest, because the force of gravity only affects the cannonball's momentum in the downward direction, and its effect is not diminished by horizontal movement. If the cannonball is launched with a greater initial horizontal velocity, then it will travel farther before it hits the ground, but it will still hit the ground in the same amount of time. However, if the cannonball is launched with an even larger initial velocity, then the curvature of the Earth becomes significant: the ground itself will curve away from the falling cannonball. A very fast cannonball will fall away from the inertial straight-line trajectory at the same rate that the Earth curves away beneath it; in other words, it will be in orbit (imagining that it is not slowed by air resistance or obstacles).[48] Main article: Harmonic oscillator An undamped spring–mass system undergoes simple harmonic motion. Consider a body of mass m {\displaystyle m} able to move along the x {\displaystyle x} axis, and suppose an equilibrium point exists at the position x = 0 {\displaystyle x=0} . That is, at x = 0 {\displaystyle x=0} , the net force upon the body is the zero vector, and by Newton's second law, the body will not accelerate. If the force upon the body is proportional to the displacement from the equilibrium point, and directed to the equilibrium point, then the body will perform simple harmonic motion. Writing the force as F = − k x {\displaystyle F=-kx} , Newton's second law becomes m d 2 x d t 2 = − k x . {\displaystyle m{\frac {d^{2}x}{dt^{2}}}=-kx\,.} This differential equation has the solution x (t) = A cos ω t + B sin ω t {\displaystyle x(t)=A\cos \omega t+B\sin \omega t\,} where the frequency ω {\displaystyle \omega } is equal to k / m {\displaystyle {\sqrt {k/m}}} , and the constants A {\displaystyle A} and B {\displaystyle B} can be calculated knowing, for example, the position and velocity the body has at a given time, like t = 0 {\displaystyle t=0} . One reason that the harmonic oscillator is a conceptually important example is that it is good approximation for many systems near a stable mechanical equilibrium.[note 10] For example, a pendulum has a stable equilibrium in the vertical position: if motionless there, it will remain there, and if pushed slightly, it will swing back and forth. Neglecting air resistance and friction in the pivot, the force upon the pendulum is gravity, and Newton's second law becomes d 2 θ d t 2 = − g L sin θ , {\displaystyle {\frac {d^{2}\theta }{dt^{2}}}=-{\frac {g}{L}}\sin \theta ,} where L {\displaystyle L} is the length of the pendulum and θ {\displaystyle \theta } is its angle from the vertical. When the angle θ {\displaystyle \theta } is small, the sine of θ {\displaystyle \theta } is nearly equal to θ {\displaystyle \theta } (see small-angle approximation), and so this expression simplifies to the equation for a simple harmonic oscillator with frequency ω = g / L {\displaystyle \omega ={\sqrt {g/L}}} . A harmonic oscillator can be damped, often by friction or viscous drag, in which case energy bleeds out of the oscillator and the amplitude of the oscillations decreases over time. Also, a harmonic oscillator can be driven by an applied force, which can lead to the phenomenon of resonance.[50] Main article: Variable-mass system Rockets, like the Space Shuttle Atlantis, expel mass during operation. This means that the mass being pushed, the rocket and its remaining onboard fuel supply, is constantly changing. Newtonian physics treats matter as being neither created nor destroyed, though it may be rearranged. It can be the case that an object of interest gains or loses mass because matter is added to or removed from it. In such a situation, Newton's laws can be applied to the individual pieces of matter, keeping track of which pieces belong to the object of interest over time. For instance, if a rocket of mass M (t) {\displaystyle M(t)} , moving at velocity v (t) {\displaystyle \mathbf {v} (t)} , ejects matter at a velocity u {\displaystyle \mathbf {u} } relative to the rocket, then[24] F = M d v d t − u d M d t {\displaystyle \mathbf {F} =M{\frac {d\mathbf {v} }{dt}}-\mathbf {u} {\frac {dM}{dt}}\,} where F {\displaystyle \mathbf {F} } is the net external force (e.g., a planet's gravitational pull).[23]:139 A boat equipped with a fan and a sail The fan and sail example is a situation studied in discussions of Newton's third law.[51] In the situation, a fan is attached to a cart or a sailboat and blows on its sail. From the third law, one would reason that the force of the air pushing in one direction would cancel out the force done by the fan on the sail, leaving the entire apparatus stationary. However, because the system is not entirely enclosed, there are conditions in which the vessel will move; for example, if the sail is built in a manner that redirects the majority of the airflow back towards the fan, the net force will result in the vessel moving forward.[34][52] The concept of energy was developed after Newton's time, but it has become an inseparable part of what is considered "Newtonian" physics. Energy can broadly be classified into kinetic, due to a body's motion, and potential, due to a body's position relative to others. Thermal energy, the energy carried by heat flow, is a type of kinetic energy not associated with the macroscopic motion of objects but instead with the movements of the atoms and molecules of which they are made. According to the work-energy theorem, when a force acts upon a body while that body moves along the line of the force, the force does work upon the body, and the amount of work done is equal to the change in the body's kinetic energy.[note 11] In many cases of interest, the net work done by a force when a body moves in a closed loop — starting at a point, moving along some trajectory, and returning to the initial point — is zero. If this is the case, then the force can be written in terms of the gradient of a function called a scalar potential:[46]:303 F = − ∇ U . {\displaystyle \mathbf {F} =-\mathbf {abla } U\,.} This is true for many forces including that of gravity, but not for friction; indeed, almost any problem in a mechanics textbook that does not involve friction can be expressed in this way.[49]:19 The fact that the force can be written in this way can be understood from the conservation of energy. Without friction to dissipate a body's energy into heat, the body's energy will trade between potential and (non-thermal) kinetic forms while the total amount remains constant. Any gain of kinetic energy, which occurs when the net force on the body accelerates it to a higher speed, must be accompanied by a loss of potential energy. So, the net force upon the body is determined by the manner in which the potential energy decreases. Main article: Rigid-body motion A rigid body is an object whose size is too large to neglect and which maintains the same shape over time. In Newtonian mechanics, the motion of a rigid body is often understood by separating it into movement of the body's center of mass and movement around the center of mass. Main article: Center of mass The total center of mass of the forks, cork, and toothpick is on top of the pen's tip. Significant aspects of the motion of an extended body can be understood by imagining the mass of that body concentrated to a single point, known as the center of mass. The location of a body's center of mass depends upon how that body's material is distributed. For a collection of pointlike objects with masses m 1 , … , m N {\displaystyle m_{1},\ldots ,m_{N}} at positions r 1 , … , r N {\displaystyle \mathbf {r} _{1},\ldots ,\mathbf {r} _{N}} , the center of mass is located at R = ∑ i = 1 N m i r i M , {\displaystyle \mathbf {R} =\sum _{i=1}^{N}{\frac {m_{i}\mathbf {r} _{i}}{M}},} where M {\displaystyle M} is the total mass of the collection. In the absence of a net external force, the center of mass moves at a constant speed in a straight line. This applies, for example, to a collision between two bodies.[55] If the total external force is not zero, then the center of mass changes velocity as though it were a point body of mass M {\displaystyle M} . This follows from the fact that the internal forces within the collection, the forces that the objects exert upon each other, occur in balanced pairs by Newton's third law. In a system of two bodies with one much more massive than the other, the center of mass will approximately coincide with the location of the more massive body.[19]:22–24 When Newton's laws are applied to rotating extended bodies, they lead to new quantities that are analogous to those invoked in the original laws. The analogue of mass is the moment of inertia, the counterpart of momentum is angular momentum, and the counterpart of force is torque. Angular momentum is calculated with respect to a reference point.[56] If the displacement vector from a reference point to a body is r {\displaystyle \mathbf {r} } and the body has momentum p {\displaystyle \mathbf {p} } , then the body's angular momentum with respect to that point is, using the vector cross product, L = r × p . {\displaystyle \mathbf {L} =\mathbf {r} \times \mathbf {p} .} Taking the time derivative of the angular momentum gives d L d t = (d r d t) × p + r × d p d t = v × m v + r × F . {\displaystyle {\frac {d\mathbf {L} }{dt}}=\left({\frac {d\mathbf {r} }{dt}}\right)\times \mathbf {p} +\mathbf {r} \times {\frac {d\mathbf {p} }{dt}}=\mathbf {v} \times m\mathbf {v} +\mathbf {r} \times \mathbf {F} .} The first term vanishes because v {\displaystyle \mathbf {v} } and m v {\displaystyle m\mathbf {v} } point in the same direction. The remaining term is the torque, τ = r × F . {\displaystyle {\boldsymbol {\tau }}=\mathbf {r} \times \mathbf {F} .} When the torque is zero, the angular momentum is constant, just as when the force is zero, the momentum is constant.[19]:14–15 The torque can vanish even when the force is non-zero, if the body is located at the reference point (r = 0 {\displaystyle \mathbf {r} =0}) or if the force F {\displaystyle \mathbf {F} } and the displacement vector r {\displaystyle \mathbf {r} } are directed along the same line. The angular momentum of a collection of point masses, and thus of an extended body, is found by adding the contributions from each of the points. This provides a means to characterize a body's rotation about an axis, by adding up the angular momenta of its individual pieces. The result depends on the chosen axis, the shape of the body, and the rate of rotation.[19]:28 Main articles: Two-body problem and Three-body problem Animation of three points or bodies attracting to each other Newton's law of universal gravitation states that any body attracts any other body along the straight line connecting them. The size of the attracting force is proportional to the product of their masses, and inversely proportional to the square of the distance between them. Finding the shape of the orbits that an inverse-square force law will produce is known as the Kepler problem. The Kepler problem can be solved in multiple ways, including by demonstrating that the Laplace–Runge–Lenz vector is constant,[57] or by applying a duality transformation to a 2-dimensional harmonic oscillator.[58] However it is solved, the result is that orbits will be conic sections, that is, ellipses (including circles), parabolas, or hyperbolas. The eccentricity of the orbit, and thus the type of conic section, is determined by the energy and the angular momentum of the orbiting body. Planets do not have sufficient energy to escape the Sun, and so their orbits are ellipses, to a good approximation; because the planets pull on one another, actual orbits are not exactly conic sections. If a third mass is added, the Kepler problem becomes the three-body problem, which in general has no exact solution in closed form. That is, there is no way to start from the differential equations implied by Newton's laws and, after a finite sequence of standard mathematical operations, obtain equations that express the three bodies' motions over time.[59][60] Numerical methods can be applied to obtain useful, albeit approximate, results for the three-body problem.[61] The positions and velocities of the bodies can be stored in variables within a computer's memory; Newton's laws are used to calculate how the velocities will change over a short interval of time, and knowing the velocities, the changes of position over that time interval can be computed. This process is looped to calculate, approximately, the bodies' trajectories. Generally speaking, the shorter the time interval, the more accurate the approximation.[62] Main article: Chaos theory Three double pendulums, initialized with almost exactly the same initial conditions, diverge over time. Newton's laws of motion allow the possibility of chaos.[63][64] That is, qualitatively speaking, physical systems obeying Newton's laws can exhibit sensitive dependence upon their initial conditions: a slight change of the position or velocity of one part of a system can lead to the whole system behaving in a radically different way within a short time. Noteworthy examples include the three-body problem, the double pendulum, dynamical billiards, and the Fermi–Pasta–Ulam–Tsingou problem. Newton's laws can be applied to fluids by considering a fluid as composed of infinitesimal pieces, each exerting forces upon neighboring pieces. The Euler momentum equation is an expression of Newton's second law adapted to fluid dynamics.[65][66] A fluid is described by a velocity field, i.e., a function v (x , t) {\displaystyle \mathbf {v} (\mathbf {x} ,t)} that assigns a velocity vector to each point in space and time. A small object being carried along by the fluid flow can change velocity for two reasons: first, because the velocity field at its position is changing over time, and second, because it moves to a new location where the velocity field has a different value. Consequently, when Newton's second law is applied to an infinitesimal portion of fluid, the acceleration a {\displaystyle \mathbf {a} } has two terms, a combination known as a total or material derivative. The mass of an infinitesimal portion depends upon the fluid density, and there is a net force upon it if the fluid pressure varies from one side of it to another. Accordingly, a = F / m {\displaystyle \mathbf {a} =\mathbf {F} /m} becomes ∂ v ∂ t + (∇ ⋅ v) v = − 1 ρ ∇ P + f , {\displaystyle {\frac {\partial v}{\partial t}}+(\mathbf {abla } \cdot \mathbf {v})\mathbf {v} =-{\frac {1}{\rho }}\mathbf {abla } P+\mathbf {f} ,} where ρ {\displaystyle \rho } is the density, P {\displaystyle P} is the pressure, and f {\displaystyle \mathbf {f} } stands for an external influence like a gravitational pull. Incorporating the effect of viscosity turns the Euler equation into a Navier–Stokes equation: ∂ v ∂ t + (∇ ⋅ v) v = − 1 ρ ∇ P + ν ∇ 2 v + f , {\displaystyle {\frac {\partial v}{\partial t}}+(\mathbf {abla } \cdot \mathbf {v})\mathbf {v} =-{\frac {1}{\rho }}\mathbf {abla } P+u abla ^{2}\mathbf {v} +\mathbf {f} ,} where ν {\displaystyle u } is the kinematic viscosity.[65] It is mathematically possible for a collection of point masses, moving in accord with Newton's laws, to launch some of themselves away so forcefully that they fly off to infinity in a finite time.[67] This unphysical behavior, known as a "noncollision singularity",[60] depends upon the masses being pointlike and able to approach one another arbitrarily closely, as well as the lack of a relativistic speed limit in Newtonian physics.[68] It is not yet known whether or not the Euler and Navier–Stokes equations exhibit the analogous behavior of initially smooth solutions "blowing up" in finite time. The question of existence and smoothness of Navier–Stokes solutions is one of the Millennium Prize Problems.[69] Classical mechanics can be mathematically formulated in multiple different ways, other than the "Newtonian" description (which itself, of course, incorporates contributions from others both before and after Newton). The physical content of these different formulations is the same as the Newtonian, but they provide different insights and facilitate different types of calculations. For example, Lagrangian mechanics helps make apparent the connection between symmetries and conservation laws, and it is useful when calculating the motion of constrained bodies, like a mass restricted to move along a curving track or on the surface of a sphere.[19]:48 Hamiltonian mechanics is convenient for statistical physics,[70][71]:57 leads to further insight about symmetry,[19]:251 and can be developed into sophisticated techniques for perturbation theory.[19]:284 Due to the breadth of these topics, the discussion here will be confined to concise treatments of how they reformulate Newton's laws of motion. Lagrangian mechanics differs from the Newtonian formulation by considering entire trajectories at once rather than predicting a body's motion at a single instant.[19]:109 It is traditional in Lagrangian mechanics to denote position with q {\displaystyle q} and velocity with q ˙ {\displaystyle {\dot {q}}} . The simplest example is a massive point particle, the Lagrangian for which can be written as the difference between its kinetic and potential energies: L (q , q ˙) = T − V , {\displaystyle L(q,{\dot {q}})=T-V,} where the kinetic energy is T = 1 2 m q ˙ 2 {\displaystyle T={\frac {1}{2}}m{\dot {q}}^{2}} and the potential energy is some function of the position, V (q) {\displaystyle V(q)} . The physical path that the particle will take between an initial point q i {\displaystyle q_{i}} and a final point q f {\displaystyle q_{f}} is the path for which the integral of the Lagrangian is "stationary". That is, the physical path has the property that small perturbations of it will, to a first approximation, not change the integral of the Lagrangian. Calculus of variations provides the mathematical tools for finding this path.[46]:485 Applying the calculus of variations to the task of finding the path yields the Euler–Lagrange equation for the particle, d d t (∂ L ∂ q ˙) = ∂ L ∂ q . {\displaystyle {\frac {d}{dt}}\left({\frac {\partial L}{\partial {\dot {q}}}}\right)={\frac {\partial L}{\partial q}}.} Evaluating the partial derivatives of the Lagrangian gives d d t (m q ˙) = − d V d q , {\displaystyle {\frac {d}{dt}}(m{\dot {q}})=-{\frac {dV}{dq}},} which is a restatement of Newton's second law. The left-hand side is the time derivative of the momentum, and the right-hand side is the force, represented in terms of the potential energy.[9]:737 Landau and Lifshitz argue that the Lagrangian formulation makes the conceptual content of classical mechanics more clear than starting with Newton's laws.[29] Lagrangian mechanics provides a convenient framework in which to prove Noether's theorem, which relates symmetries and conservation laws.[72] The conservation of momentum can be derived by applying Noether's theorem to a Lagrangian for a multi-particle system, and so, Newton's third law is a theorem rather than an assumption.[19]:124 Emmy Noether, whose 1915 proof of a celebrated theorem that relates symmetries and conservation laws was a key development in modern physics and can be conveniently stated in the language of Lagrangian or Hamiltonian mechanics In Hamiltonian mechanics, the dynamics of a system are represented by a function called the Hamiltonian, which in many cases of interest is equal to the total energy of the system.[9]:742 The Hamiltonian is a function of the positions and the momenta of all the bodies making up the system, and it may also depend explicitly upon time. The time derivatives of the position and momentum variables are given by partial derivatives of the Hamiltonian, via Hamilton's equations.[19]:203 The simplest example is a point mass m {\displaystyle m} constrained to move in a straight line, under the effect of a potential. Writing q {\displaystyle q} for the position coordinate and p {\displaystyle p} for the body's momentum, the Hamiltonian is H (p , q) = p 2 2 m + V (q) . {\displaystyle {\mathcal {H}}(p,q)={\frac {p^{2}}{2m}}+V(q).} In this example, Hamilton's equations are d q d t = ∂ H ∂ p {\displaystyle {\frac {dq}{dt}}={\frac {\partial {\mathcal {H}}}{\partial p}}} and d p d t = − ∂ H ∂ q . {\displaystyle {\frac {dp}{dt}}=-{\frac {\partial {\mathcal {H}}}{\partial q}}.} Evaluating these partial derivatives, the former equation becomes d q d t = p m , {\displaystyle {\frac {dq}{dt}}={\frac {p}{m}},} which reproduces the familiar statement that a body's momentum is the product of its mass and velocity. The time derivative of the momentum is d p d t = − d V d q , {\displaystyle {\frac {dp}{dt}}=-{\frac {dV}{dq}},} which, upon identifying the negative derivative of the potential with the force, is just Newton's second law once again.[66][9]:742 As in the Lagrangian formulation, in Hamiltonian mechanics the conservation of momentum can be derived using Noether's theorem, making Newton's third law an idea that is deduced rather than assumed.[19]:251 Among the proposals to reform the standard introductory-physics curriculum is one that teaches the concept of energy before that of force, essentially "introductory Hamiltonian mechanics".[73][74] The Hamilton–Jacobi equation provides yet another formulation of classical mechanics, one which makes it mathematically analogous to wave optics.[19]:284[75] This formulation also uses Hamiltonian functions, but in a different way than the formulation described above. The paths taken by bodies or collections of bodies are deduced from a function S (q 1 , q 2 , … , t) {\displaystyle S(\mathbf {q} _{1},\mathbf {q} _{2},\ldots ,t)} of positions q i {\displaystyle \mathbf {q} _{i}} and time t {\displaystyle t} . The Hamiltonian is incorporated into the Hamilton–Jacobi equation, a differential equation for S {\displaystyle S} . Bodies move over time in such a way that their trajectories are perpendicular to the surfaces of constant S {\displaystyle S} , analogously to how a light ray propagates in the direction perpendicular to its wavefront. This is simplest to express for the case of a single point mass, in which S {\displaystyle S} is a function S (q , t) {\displaystyle S(\mathbf {q} ,t)} , and the point mass moves in the direction along which S {\displaystyle S} changes most steeply. In other words, the momentum of the point mass is the gradient of S {\displaystyle S} : v = 1 m ∇ S . {\displaystyle \mathbf {v} ={\frac {1}{m}}\mathbf {\nabla } S.} The Hamilton–Jacobi equation for a point mass is − ∂ S ∂ t = H (q , ∇ S , t) . {\displaystyle -{\frac {\partial S}{\partial t}}=H\left(\mathbf {q} ,\mathbf {\nabla } S,t\right).} The relation to Newton's laws can be seen by considering a point mass moving in a time-independent potential V (q) {\displaystyle V(\mathbf {q})} , in which case the Hamilton–Jacobi equation becomes − ∂ S ∂ t = 1 2 m (∇ S) 2 + V (q) . {\displaystyle -{\frac {\partial S}{\partial t}}={\frac {1}{2m}}\left(\mathbf {\nabla } S\right)^{2}+V(\mathbf {q}).} Taking the gradient of both sides, this becomes − ∇ ∂ S ∂ t = 1 2 m ∇ (∇ S) 2 + ∇ V . {\displaystyle -\mathbf {\nabla } {\frac {\partial S}{\partial t}}={\frac {1}{2m}}\mathbf {\nabla } \left(\mathbf {\nabla } S\right)^{2}+\mathbf {\nabla } V.} Interchanging the order of the partial derivatives on the left-hand side, and using the power and chain rules on the first term on the right-hand side, − ∂ ∂ t ∇ S = 1 m (∇ S ⋅ ∇) ∇ S + ∇ V . {\displaystyle -{\frac {\partial }{\partial t}}\mathbf {\nabla } S={\frac {1}{m}}\left(\mathbf {\nabla } S\cdot \mathbf {\nabla } \right)\mathbf {\nabla } S+\mathbf {\nabla } V.} Gathering together the terms that depend upon the gradient of S {\displaystyle S} , [∂ ∂ t + 1 m (∇ S ⋅ ∇)] ∇ S = − ∇ V . {\displaystyle \left[{\frac {\partial }{\partial t}}+{\frac {1}{m}}\left(\mathbf {\nabla } S\cdot \mathbf {\nabla } \right)\right]\mathbf {\nabla } S=-\mathbf {\nabla } V.} This is another re-expression of Newton's second law.[76] The expression in brackets is a total or material derivative as mentioned above,[77] in which the first term indicates how the function being differentiated changes over time at a fixed location, and the second term captures how a moving particle will see different values of that function as it travels from place to place: [∂ ∂ t + 1 m (∇ S ⋅ ∇)] = [∂ ∂ t + v ⋅ ∇] = d d t . {\displaystyle \left[{\frac {\partial }{\partial t}}+{\frac {1}{m}}\left(\mathbf {\nabla } S\cdot \mathbf {\nabla } \right)\right]=\left[{\frac {\partial }{\partial t}}+\mathbf {v} \cdot \mathbf {\nabla } \right]={\frac {d}{dt}}.} A simulation of a larger, but still microscopic, particle (in yellow) surrounded by a gas of smaller particles, illustrating Brownian motion In statistical physics, the kinetic theory of gases applies Newton's laws of motion to large numbers (typically on the order of the Avogadro number) of particles. Kinetic theory can explain, for example, the pressure that a gas exerts upon the container holding it as the aggregate of many impacts of atoms, each imparting a tiny amount of momentum.[71]:62 The Langevin equation is a special case of Newton's second law, adapted for the case of describing a small object bombarded stochastically by even smaller ones.[78]:235 It can be written m a = − γ v + ξ {\displaystyle m\mathbf {a} =-\gamma \mathbf {v} +\mathbf {\xi } \,} where γ {\displaystyle \gamma } is a drag coefficient and ξ {\displaystyle \mathbf {\xi } } is a force that varies randomly from instant to instant, representing the net effect of collisions with the surrounding particles. This is used to model Brownian motion.[79] Newton's three laws can be applied to phenomena involving electricity and magnetism, though subtleties and caveats exist. Coulomb's law for the electric force between two stationary, electrically charged bodies has much the same mathematical form as Newton's law of universal gravitation: the force is proportional to the product of the charges, inversely proportional to the square of the distance between them, and directed along the straight line between them. The Coulomb force that a charge q 1 {\displaystyle q_{1}} exerts upon a charge q 2 {\displaystyle q_{2}} is equal in magnitude to the force that q 2 {\displaystyle q_{2}} exerts upon q 1 {\displaystyle q_{1}} , and it points in the exact opposite direction. Coulomb's law is thus consistent with Newton's third law.[80] Electromagnetism treats forces as produced by fields acting upon charges. The Lorentz force law provides an expression for the force upon a charged body that can be plugged into Newton's second law in order to calculate its acceleration.[81]:85 According to the Lorentz force law, a charged body in an electric field experiences a force in the direction of that field, a force proportional to its charge q {\displaystyle q} and to the strength of the electric field. In addition, a moving charged body in a magnetic field experiences a force that is also proportional to its charge, in a direction perpendicular to both the field and the body's direction of motion. Using the vector cross product, F = q E + q v × B . {\displaystyle \mathbf {F} =q\mathbf {E} +q\mathbf {v} \times \mathbf {B} .} The Lorentz force law in effect: electrons are bent into a circular trajectory by a magnetic field. If the electric field vanishes (E = 0 {\displaystyle \mathbf {E} =0}), then the force will be perpendicular to the charge's motion, just as in the case of uniform circular motion studied above, and the charge will circle (or more generally move in a helix) around the magnetic field lines at the cyclotron frequency ω = q B / m {\displaystyle \omega =qB/m} .[78]:222 Mass spectrometry works by applying electric and/or magnetic fields to moving charges and measuring the resulting acceleration, which by the Lorentz force law yields the mass-to-charge ratio.[82] Collections of charged bodies do not always obey Newton's third law: there can be a change of one body's momentum without a compensatory change in the momentum of another. The discrepancy is accounted for by momentum carried by the electromagnetic field itself. The momentum per unit volume of the electromagnetic field is proportional to the Poynting vector.[83]:184[84] There is subtle conceptual conflict between electromagnetism and Newton's first law: Maxwell's theory of electromagnetism predicts that electromagnetic waves will travel through empty space at a constant, definite speed. Thus, some inertial observers seemingly have a privileged status over the others, namely those who measure the speed of light and find it to be the value predicted by the Maxwell equations. In other words, light provides an absolute standard for speed, yet the principle of inertia holds that there should be no such standard. This tension is resolved in the theory of special relativity, which revises the notions of space and time in such a way that all inertial observers will agree upon the speed of light in vacuum.[note 12] Further information: Relativistic mechanics and Acceleration (special relativity) In special relativity, the rule that Wilczek called "Newton's Zeroth Law" breaks down: the mass of a composite object is not merely the sum of the masses of the individual pieces.[87]:33 Newton's first law, inertial motion, remains true. A form of Newton's second law, that force is the rate of change of momentum, also holds, as does the conservation of momentum. However, the definition of momentum is modified. Among the consequences of this is the fact that the more quickly a body moves, the harder it is to accelerate, and so, no matter how much force is applied, a body cannot be accelerated to the speed of light. Depending on the problem at hand, momentum in special relativity can be represented as a three-dimensional vector, p = m γ v {\displaystyle \mathbf {p} =m\gamma \mathbf {v} } , where m {\displaystyle m} is the body's rest mass and γ {\displaystyle \gamma } is the Lorentz factor, which depends upon the body's speed. Alternatively, momentum and force can be represented as four-vectors.[88]:107 Newton's third law must be modified in special relativity. The third law refers to the forces between two bodies at the same moment in time, and a key feature of special relativity is that simultaneity is relative. Events that happen at the same time relative to one observer can happen at different times relative to another. So, in a given observer's frame of reference, action and reaction may not be exactly opposite, and the total momentum of interacting bodies may not be conserved. The conservation of momentum is restored by including the momentum stored in the field that describes the bodies' interaction.[89][90] Newtonian mechanics is a good approximation to special relativity when the speeds involved are small compared to that of light.[91]:131 General relativity is a theory of gravity that advances beyond that of Newton. In general relativity, the gravitational force of Newtonian mechanics is reimagined as curvature of spacetime. A curved path like an orbit, attributed to a gravitational force in Newtonian mechanics, is not the result of a force deflecting a body from an ideal straight-line path, but rather the body's attempt to fall freely through a background that is itself curved by the presence of other masses. A remark by John Archibald Wheeler that has become proverbial among physicists summarizes the theory: "Spacetime tells matter how to move; matter tells spacetime how to curve."[92][93] Wheeler himself thought of this reciprocal relationship as a modern, generalized form of Newton's third law.[92] The relation between matter distribution and spacetime curvature is given by the Einstein field equations, which require tensor calculus to express.[87]:43[94] The Newtonian theory of gravity is a good approximation to the predictions of general relativity when gravitational effects are weak and objects are moving slowly compared to the speed of light.[85]:327[95] Quantum mechanics is a theory of physics originally developed in order to understand microscopic phenomena: behavior at the scale of molecules, atoms or subatomic particles. Generally and loosely speaking, the smaller a system is, the more an adequate mathematical model will require understanding quantum effects. The conceptual underpinning of quantum physics is very different from that of classical physics. Instead of thinking about quantities like position, momentum, and energy as properties that an object has, one considers what result might appear when a measurement of a chosen type is performed. Quantum mechanics allows the physicist to calculate the probability that a chosen measurement will elicit a particular result.[96][97] The expectation value for a measurement is the average of the possible results it might yield, weighted by their probabilities of occurrence.[98] The Ehrenfest theorem provides a connection between quantum expectation values and Newton's second law, a connection that is necessarily inexact, as quantum physics is fundamentally different from classical. In quantum physics, position and momentum are represented by mathematical entities known as Hermitian operators, and the Born rule is used to calculate the expectation values of a position measurement or a momentum measurement. These expectation values will generally change over time; that is, depending on the time at which (for example) a position measurement is performed, the probabilities for its different possible outcomes will vary. The Ehrenfest theorem says, roughly speaking, that the equations describing how these expectation values change over time have a form reminiscent of Newton's second law. However, the more pronounced quantum effects are in a given situation, the more difficult it is to derive meaningful conclusions from this resemblance.[note 13] Isaac Newton (1643–1727), in a 1689 portrait by Godfrey Kneller Newton's own copy of his Principia, with hand-written corrections for the second edition, in the Wren Library at Trinity College, Cambridge Newton's first and second laws, in Latin, from the original 1687 Principia Mathematica The concepts invoked in Newton's laws of motion — mass, velocity, momentum, force — have predecessors in earlier work, and the content of Newtonian physics was further developed after Newton's time. Newton combined knowledge of celestial motions with the study of events on Earth and showed that one theory of mechanics could encompass both.[note 14] As noted by scholar I. Bernard Cohen, Newton's work was more than a mere synthesis of previous results, as he selected certain ideas and further transformed them, with each in a new form that was useful to him, while at the same time proving false of certain basic or fundamental principles of scientists such as Galileo Galilei, Johannes Kepler, René Descartes, and Nicolaus Copernicus.[103] He approached natural philosophy with mathematics in a completely novel way, in that instead of a preconceived natural philosophy, his style was to begin with a mathematical construct, and build on from there, comparing it to the real world to show that his system accurately accounted for it.[104] Aristotle (384–322 BCE) The subject of physics is often traced back to Aristotle, but the history of the concepts involved is obscured by multiple factors. An exact correspondence between Aristotelian and modern concepts is not simple to establish: Aristotle did not clearly distinguish what we would call speed and force, used the same term for density and viscosity, and conceived of motion as always through a medium, rather than through space. In addition, some concepts often termed "Aristotelian" might better be attributed to his followers and commentators upon him.[105] These commentators found that Aristotelian physics had difficulty explaining projectile motion.[note 15] Aristotle divided motion into two types: "natural" and "violent". The "natural" motion of terrestrial solid matter was to fall downwards, whereas a "violent" motion could push a body sideways. Moreover, in Aristotelian physics, a "violent" motion requires an immediate cause; separated from the cause of its "violent" motion, a body would revert to its "natural" behavior. Yet, a javelin continues moving after it leaves the thrower's hand. Aristotle concluded that the air around the javelin must be imparted with the ability to move the javelin forward. John Philoponus, a Byzantine Greek thinker active during the sixth century, found this absurd: the same medium, air, was somehow responsible both for sustaining motion and for impeding it. If Aristotle's idea were true, Philoponus said, armies would launch weapons by blowing upon them with bellows. Philoponus argued that setting a body into motion imparted a quality, impetus, that would be contained within the body itself. As long as its impetus was sustained, the body would continue to move.[107]:47 In the following centuries, versions of impetus theory were advanced by individuals including Nur ad-Din al-Bitruji, Avicenna, Abu'l-Barakāt al-Baghdādī, John Buridan, and Albert of Saxony. In retrospect, the idea of impetus can be seen as a forerunner of the modern concept of momentum.[note 16] The intuition that objects move according to some kind of impetus persists in many students of introductory physics.[109] See also: Galileo Galilei § Inertia The French philosopher René Descartes introduced the concept of inertia by way of his "laws of nature" in The World (Traité du monde et de la lumière) written 1629–33. However, The World purported a heliocentric worldview, and in 1633 this view had given rise a great conflict between Galileo Galilei and the Roman Catholic Inquisition. Descartes knew about this controversy and did not wish to get involved. The World was not published until 1664, ten years after his death.[110] Galileo Galilei (1564–1642) The modern concept of inertia is credited to Galileo. Based on his experiments, Galileo concluded that the "natural" behavior of a moving body was to keep moving, until something else interfered with it. In Two New Sciences (1638) Galileo wrote:[111][112]Imagine any particle projected along a horizontal plane without friction; then we know, from what has been more fully explained in the preceding pages, that this particle will move along this same plane with a motion which is uniform and perpetual, provided the plane has no limits.René Descartes (1596–1650)Galileo recognized that in projectile motion, the Earth's gravity affects vertical but not horizontal motion.[113] However, Galileo's idea of inertia was not exactly the one that would be codified into Newton's first law. Galileo thought that a body moving a long distance inertially would follow the curve of the Earth. This idea was corrected by Isaac Beeckman, Descartes, and Pierre Gassendi, who recognized that inertial motion should be motion in a straight line.[114] Descartes published his laws of nature (laws of motion) with an explanation of the concepts in Principles of Philosophy (Principia Philosophiae) in 1644, with the heliocentric part toned down.[115][110] Ball in circular motion has string cut and flies off tangentially. First Law of Nature: Each thing when left to itself continues in the same state; so any moving body goes on moving until something stops it.Second Law of Nature: Each moving thing if left to itself moves in a straight line; so any body moving in a circle always tends to move away from the centre of the circle. According to American philosopher Richard J. Blackwell, Dutch scientist Christiaan Huygens had worked out his own, concise version of the law in 1656,[116] it was not published until 1703, eight years after his death, in the opening paragraph of De Motu Corporum ex Percussione. Hypothesis I: Any body already in motion will continue to move perpetually with the same speed and in a straight line unless it is impeded. According to Huygens, this law was already known by Galileo and Descartes among others.[116] Christiaan Huygens (1629–1695) Christiaan Huygens, in his Horologium Oscillatorium (1673), put forth the hypothesis that "By the action of gravity, whatever its sources, it happens that bodies are moved by a motion composed both of a uniform motion in one direction or another and of a motion downward due to gravity." Newton's second law generalized this hypothesis from gravity to all forces.[117] One important characteristic of Newtonian physics is that forces can act at a distance without requiring physical contact.[note 17] For example, the Sun and the Earth pull on each other gravitationally, despite being separated by millions of kilometres. This contrasts with the idea, championed by Descartes among others, that the Sun's gravity held planets in orbit by swirling them in a vortex of transparent matter, aether.[118] Newton considered aetherial explanations of force but ultimately rejected them.[122] The study of magnetism by William Gilbert and others created a precedent for thinking of immaterial forces,[122] and unable to find a quantitatively satisfactory explanation of his law of gravity in terms of an aetherial model, Newton eventually declared, "I feign no hypotheses": whether or not a model like Descartes's vortices could be found to underlie the Principia's theories of motion and gravity, the first grounds for judging them must be the successful predictions they made.[123] And indeed, since Newton's time every attempt at such a model has failed. Johannes Kepler (1571–1630) Johannes Kepler suggested that gravitational attractions were reciprocal — that, for example, the Moon pulls on the Earth while the Earth pulls on the Moon — but he did not argue that such pairs are equal and opposite.[126] In his Principles of Philosophy (1644), Descartes introduced the idea that during a collision between bodies, a "quantity of motion" remains unchanged. Descartes defined this quantity somewhat imprecisely by adding up the products of the speed and "size" of each body, where "size" for him incorporated both volume and surface area.[127] Moreover, Descartes thought of the universe as a plenum, that is, filled with matter, so all motion required a body to displace a medium as it moved. During the 1650s, Huygens studied collisions between hard spheres and deduced a principle that is now identified as the conservation of momentum.[128][129] Christopher Wren would later deduce the same rules for elastic collisions that Huygens had, and John Wallis would apply momentum conservation to study inelastic collisions. Newton cited the work of Huygens, Wren, and Wallis to support the validity of his third law.[130] Newton arrived at his set of three laws incrementally. In a 1684 manuscript written to Huygens, he listed four laws: the principle of inertia, the change of motion by force, a statement about relative motion that would today be called Galilean invariance, and the rule that interactions between bodies do not change the motion of their center of mass. In a later manuscript, Newton added a law of action and reaction, while saying that this law and the law regarding the center of mass implied one another. Newton probably settled on the presentation in the Principia, with three primary laws and then other statements reduced to corollaries, during 1685.[131] Page 157 from Mechanism of the Heavens (1831), Mary Somerville's expanded version of the first two volumes of Laplace's Traité de mécanique céleste.[132] Here, Somerville deduces the inverse-square law of gravity from Kepler's laws of planetary motion. Newton expressed his second law by saying that the force on a body is proportional to its change of motion, or momentum. By the time he wrote the Principia, he had already developed calculus (which he called "the science of fluxions"), but in the Principia he made no explicit use of it, perhaps because he believed geometrical arguments in the tradition of Euclid to be more rigorous.[133]:15[134] Consequently, the Principia does not express acceleration as the second derivative of position, and so it does not give the second law as F = m a {\displaystyle F=ma} . This form of the second law was written (for the special case of constant force) at least as early as 1716, by Jakob Hermann; Leonhard Euler would employ it as a basic premise in the 1740s.[135] Euler pioneered the study of rigid bodies[136] and established the basic theory of fluid dynamics.[137] Pierre-Simon Laplace's five-volume Traité de mécanique céleste (1798–1825) forsook geometry and developed mechanics purely through algebraic expressions, while resolving questions that the Principia had left open, like a full theory of the tides.[138] The concept of energy became a key part of Newtonian mechanics in the post-Newton period. Huygens' solution of the collision of hard spheres showed that in that case, not only is momentum conserved, but kinetic energy is as well (or, rather, a quantity that in retrospect we can identify as one-half the total kinetic energy). The question of what is conserved during all other processes, like inelastic collisions and motion slowed by friction, was not resolved until the 19th century. Debates on this topic overlapped with philosophical disputes between the metaphysical views of Newton and Leibniz, and variants of the term "force" were sometimes used to denote what we would call types of energy. For example, in 1742, Émilie du Châtelet wrote: "Dead force consists of a simple tendency to motion: such is that of a spring ready to relax; living force is that which a body has when it is in actual motion." In modern terminology, "dead force" and "living force" correspond to potential energy and kinetic energy respectively.[139] Conservation of energy was not established as a universal principle until it was understood that the energy of mechanical work can be dissipated into heat.[140][141] With the concept of energy given a solid grounding, Newton's laws could then be derived within formulations of classical mechanics that put energy first, as in the Lagrangian and Hamiltonian formulations described above. Modern presentations of Newton's laws use the mathematics of vectors, a topic that was not developed until the late 19th and early 20th centuries. Vector algebra, pioneered by Josiah Willard Gibbs and Oliver Heaviside, stemmed from and largely supplanted the earlier system of quaternions invented by William Rowan Hamilton.[142][143] Euler's laws of motion History of classical mechanics List of eponymous laws List of equations in classical mechanics List of scientific laws named after people List of textbooks on classical mechanics and quantum mechanics Norton's dome ^ See, for example, Zain.[4]:1-2 David Tong observes, "A particle is defined to be an object of insignificant size: e.g. an electron, a tennis ball or a planet. Obviously the validity of this statement depends on the context..."[5] ^ Negative acceleration includes both slowing down (when the current velocity is positive) and speeding up (when the current velocity is negative). For this and other points that students have often found difficult, see McDermott et al.[8] ^ Per Cohen and Whitman.[2] For other phrasings, see Eddington[14] and Frautschi et al.[15]:114 Andrew Motte's 1729 translation rendered Newton's "nisi quatenus" as unless instead of except insofar, which Hoek argues was erroneous.[16][17] ^ One textbook observes that a block sliding down an inclined plane is what "some cynics view as the dullest problem in all of physics".[23]:70 Another quips, "Nobody will ever know how many minds, eager to learn the secrets of the universe, found themselves studying inclined planes and pulleys instead, and decided to switch to some more interesting profession."[15]:173 ^ For example, José and Saletan (following Mach and Eisenbud[27]) take the conservation of momentum as a fundamental physical principle and treat F = m a {\displaystyle \mathbf {F} =m\mathbf {a} } as a definition of "force".[19]:9 See also Frautschi et al.[15]:134 as well as Feynman, Leighton and Sands,[28]:12-1 who argue that the second law is incomplete without a specification of a force by another law, like the law of gravity. Kleppner and Kolenkow argue that the second law is incomplete without the third law: an observer who sees one body accelerate without a matching acceleration of some other body to compensate would conclude, not that a force is acting, but that they are not an inertial observer.[23]:60 Landau and Lifshitz bypass the question by starting with the Lagrangian formalism rather than the Newtonian.[29] ^ See, for instance, Moebs et al.,[31] Gonick and Huffman,[32] Low and Wilson,[33] Stocklmayer et al.,[34] Hellingman,[35] and Hodanbosi.[36] ^ See, for example, Frautschi et al.[15]:356 ^ For the former, see Greiner,[39] or Wachter and Hoeber.[40] For the latter, see Tait[41] and Heaviside.[42] ^ Among the many textbook explanations of this are Frautschi et al.[15]:104 and Boas.[46]:287 ^ Among the many textbook treatments of this point are Hand and Finch[49]:81 and also Kleppner and Kolenkow.[23]:103 ^ Treatments can be found in, e.g., Chabay et al.[53] and McCallum et al.[54]:449 ^ Discussions can be found in, for example, Frautschi et al.,[15]:215 Panofsky and Phillips,[83]:272 Goldstein, Poole and Safko,[85]:277 and Werner.[86] ^ Details can be found in the textbooks by, e.g., Cohen-Tannoudji et al.[99]:242 and Peres.[100]:302 ^ As one physicist writes, "Physical theory is possible because we are immersed and included in the whole process – because we can act on objects around us. Our ability to intervene in nature clarifies even the motion of the planets around the sun – masses so great and distances so vast that our roles as participants seem insignificant. Newton was able to transform Kepler's kinematical description of the solar system into a far more powerful dynamical theory because he added concepts from Galileo's experimental methods – force, mass, momentum, and gravitation. The truly external observer will only get as far as Kepler. Dynamical concepts are formulated on the basis of what we can set up, control, and measure."[101] See, for example, Caspar and Hellman.[102] ^ Aristotelian physics also had difficulty explaining buoyancy, a point that Galileo tried to resolve without complete success.[106] ^ Anneliese Maier cautions, "Impetus is neither a force, nor a form of energy, nor momentum in the modern sense; it shares something with all these other concepts, but it is identical with none of them."[108]:79 ^ Newton himself was an enthusiastic alchemist. John Maynard Keynes called him "the last of the magicians" to describe his place in the transition between protoscience and modern science.[118][119] The suggestion has been made that alchemy inspired Newton's notion of "action at a distance", i.e., one body exerting a force upon another without being in direct contact.[120] This suggestion enjoyed considerable support among historians of science[121] until a more extensive study of Newton's papers became possible, after which it fell out of favor. However, it does appear that Newton's alchemy influenced his optics, in particular, how he thought about the combination of colors.[122][123]

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